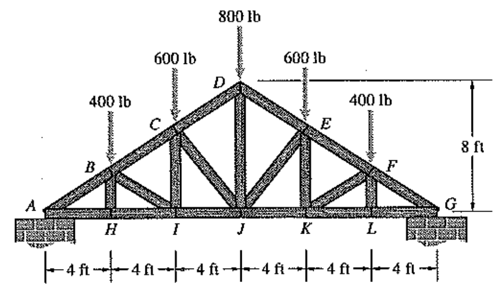
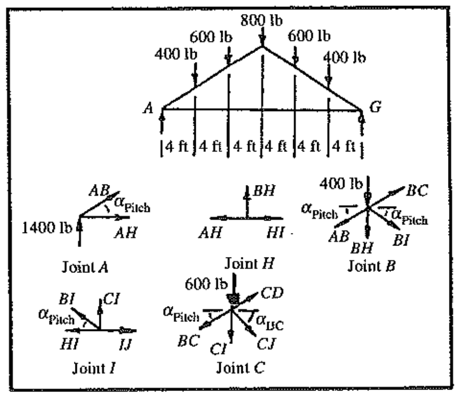
Problem 6.26 The Howe truss helps support a roof. Model the supports at A and G as roller supports. Determine the axial forces in members AB, BC, and CD.



Solution: The strategy is to proceed from end A, choosing joints with only one unknown axial force in the x- and/or y-direction, if possible, and if not, establish simultaneous conditions in the unknowns.



The interior angles HIB and HJC differ. The pitch angle is

$$\alpha_{\text{Pitch}} = \tan^{-1}\left(\frac{8}{12}\right) = 33.7^\circ.$$

The length of the vertical members:

$$\overline{BH} = 4\left(\frac{8}{12}\right) = 2.6667 \text{ ft},$$

from which the angle

$$\alpha_{HIB} = \tan^{-1}\left(\frac{2.6667}{4}\right) = 33.7^\circ.$$

$$\overline{CI} = 8\frac{8}{12} = 5.3333 \text{ ft},$$

from which the angle

$$\alpha_{IJC} = \tan^{-1}\left(\frac{5.333}{4}\right) = 53.1^\circ.$$

The moment about G:

$$M_G = (4+20)(400) + (8+16)(600) + (12)(800) - 24A = 0,$$

from which $A = \frac{33600}{24} = 1400$ lb. *Check*: The total load is 2800 lb. From left-right symmetry each support A, G supports half the total load. *check*.

The method of joints: Denote the axial force in a member joining two points I, K by IK.

Joint A:

$$\sum F_y = AB \sin\alpha_P + 1400 = 0,$$

from which $\boxed{AB = -\frac{1400}{\sin\alpha_p} = -2523.9 \text{ lb } (C)}$

$$\sum F_x = AB\cos\alpha_{\text{Pitch}} + AH = 0,$$

from which $AH = (2523.9)(0.8321) = 2100$ lb (T)

Joint H:

$$\sum F_y = BH = 0, \text{ or, } BH = 0.$$

$$\sum F_x = -AH + HI = 0,$$

from which $HI = 2100$ lb (T)

Joint B:

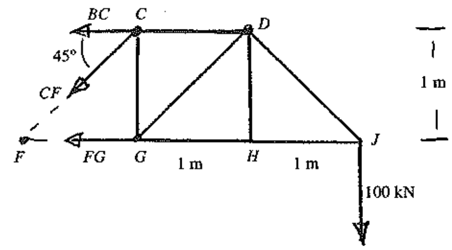
$$\sum F_x = -AB\cos\alpha_{\text{Pitch}} + BC\cos\alpha_{\text{Pitch}}$$

$$+ BI\cos\alpha_{\text{Pitch}} = 0,$$

from which $BC + BI = AB$

Problem 6.35 For the truss in Problem 6.34, use the method of sections to determine the axial forces in members BC, CF, and FG.

Solution:



$\sum F_x$: $-BC - CF\cos 45 - FG = 0$

$\sum F_y$: $-CF\sin 45° - 100 = 0$

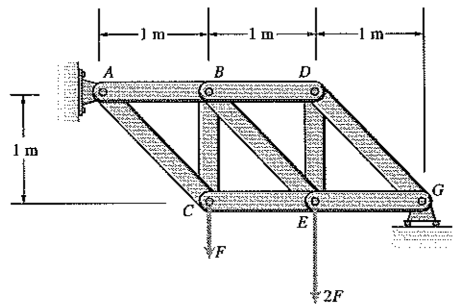
$\sum M_C$: $-(1)FG - 2(100) = 0$

Solving $BC = 300$ kN (T)

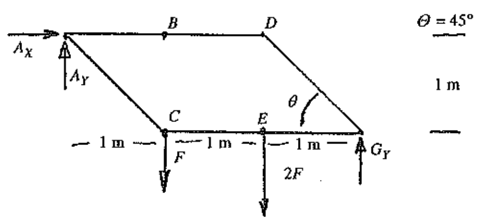
$CF = -141.4$ kN (C)

$FG = -200$ kN (C)

Problem 6.36 Use the method of sections to determine the axial forces in members AB, BC, and CE.



Solution: First, determine the forces at the supports



$\sum F_x$: $A_x = 0$

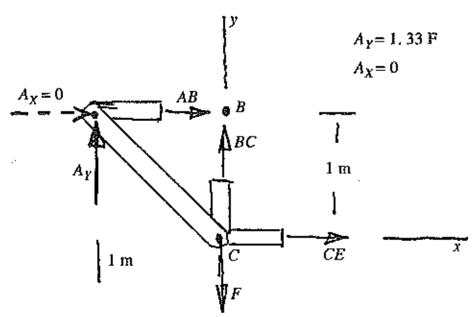
$\sum F_y$: $A_y + G_y - 3F = 0$

$+\sum M_A$: $-1(F) - 2(2F) + 3G_y = 0$

Solving $A_x = 0$ $G_y = 1.67F$

$A_y = 1.33F$

Method of Sections:



$\sum F_x$: $CE + AB = 0$

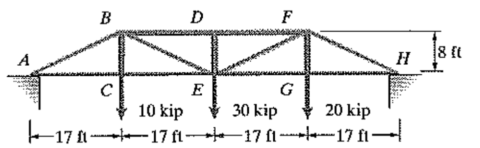
$\sum F_y$: $BC + A_y - F = 0$

$+\sum M_B$: $(-1)A_y + (1)CE = 0$

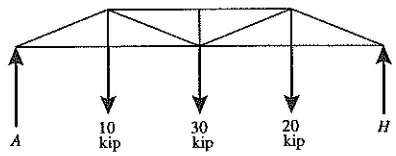
Solving, we get

$AB = -1.33F$ (C)
$CE = 1.33F$ (T)
$BC = -0.33F$ (C)

Problem 6.38 The Pratt bridge truss is loaded as shown. Use the method of sections to determine the axial forces in members *BD*, *BE*, and *CE*.



Solution: Use the whole structure to find the reaction at *A*.

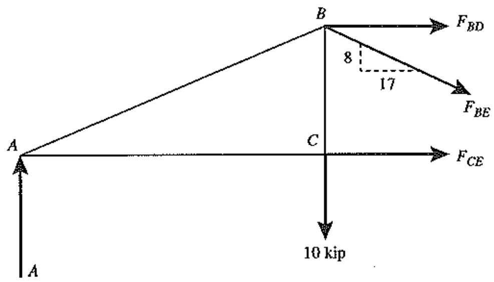


$$\sum M_H : (20 \text{ kip})(17 \text{ ft}) + (30 \text{ kip})(34 \text{ ft})$$

$$+ (10 \text{ kip})(51 \text{ ft}) - A(68 \text{ ft}) = 0$$

$$\Rightarrow A = 27.5 \text{ kip}$$

Now cut through *BD*, *BE*, *CE* and use the left section



$$\sum M_B : -A(17 \text{ ft}) + F_{CE}(8 \text{ ft}) = 0 \Rightarrow F_{CE} = 58.4 \text{ kip}$$

$$\sum M_E : (10 \text{ kip})(17 \text{ ft}) - A(34 \text{ ft}) - F_{BD}(8 \text{ ft}) = 0$$

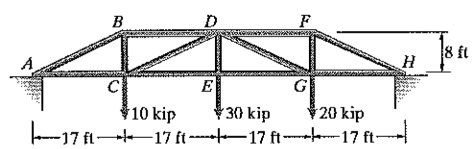
$$\Rightarrow F_{BD} = -95.6 \text{ kip}$$

$$\sum F_y : A - 10 \text{ kip} - \frac{8}{\sqrt{353}} F_{BE} = 0 \Rightarrow F_{BE} = 41.1 \text{ kip}$$

In Summary

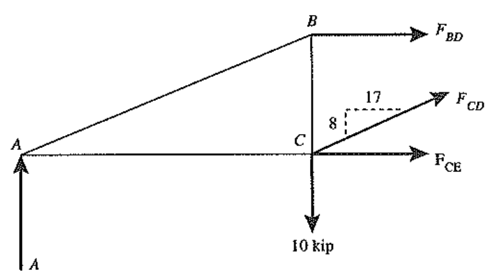
$$\boxed{F_{CE} = 58.4 \text{ kip}(T),\ F_{BD} = 95.6 \text{ kip}(C),\ F_{BE} = 41.1 \text{ kip}(T)}$$

Problem 6.39 The Howe bridge truss is loaded as shown. Use the method of sections to determine the axial forces in members *BD*, *CD*, and *CE*.



Solution: Use the whole structure to find the reaction at *A* (same as 6.38) $A = 27.5$ kip

Now cut through *BD*, *CD*, and *CE* and use the left section.



$$\sum M_C : -A(17 \text{ ft}) - F_{BD}(8 \text{ ft}) = 0 \Rightarrow F_{BD} = -58.4 \text{ kip}$$

$$\sum M_D : -A(34 \text{ ft}) + (10 \text{ kip})(17 \text{ ft}) + F_{CE}(8 \text{ ft}) = 0$$

$$\Rightarrow F_{CE} = 95.6 \text{ kip}$$

$$\sum F_y : A - 10 \text{ kip} + \frac{8}{\sqrt{353}} F_{CD} = 0 \Rightarrow F_{CD} = -41.1 \text{ kip}$$

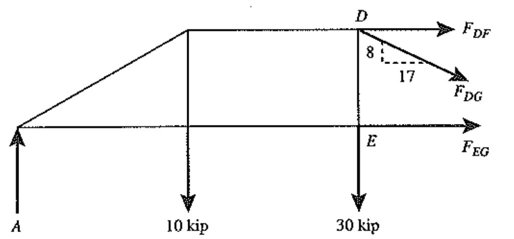
In Summary

$$\boxed{F_{BD} = 58.4 \text{ kip}(C),\ F_{CE} = 95.6 \text{ kip}(T),\ F_{CD} = 41.1 \text{ kip}(C)}$$

Problem 6.40 For the Howe bridge truss in Problem 6.39, use the method of sections to determine the axial forces in members DF, DG, and EG.

Solution: Same truss as 6.39.

Cut through DF, DG, and EG and use left section



$$\sum M_D : -A(34\text{ ft}) + (10\text{ kip})(17\text{ ft}) + F_{EG}(8\text{ ft}) = 0$$

$$\Rightarrow F_{EG} = 95.6\text{ kip}$$

$$\sum M_G : -A(51\text{ ft}) + (10\text{ kip})(34\text{ ft}) + (30\text{ kip})(17\text{ ft}) - F_{DF}(8\text{ ft})$$

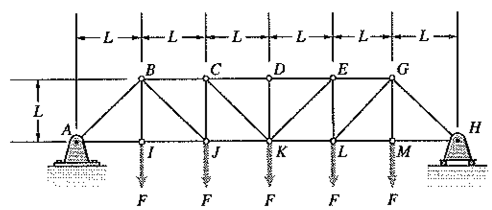
$$= 0 \Rightarrow F_{DF} = -69.1\text{ kip}$$

$$\sum F_y : A - 10\text{ kip} - 30\text{ kip} - \frac{8}{\sqrt{353}} F_{DG} = 0 \Rightarrow F_{DG} = -29.4\text{ kip}$$

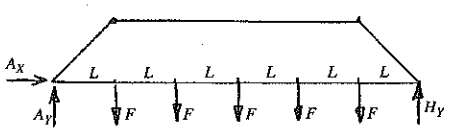
In summary

$$\boxed{F_{EG} = 95.6\text{ kip}(T),\ F_{DF} = 69.1\text{ kip}(C),\ F_{DG} = 29.4\text{ kip}(C)}$$

Problem 6.41 The Pratt bridge truss supports five forces $F = 340$ kN. The dimension $L = 8$ m. Use the method of sections to determine the axial force in member JK.



Solution: First determine the external support forces.



$F = 340$ kN, $L = 8$ M

$$\sum F_x: \quad A_x = 0$$

$$\sum F_y: \quad A_y - 5F + H_y = 0$$

$$+\sum M_A: \quad 6LH_y - LF - 2LF - 3LF - 4LF - 5LF = 0$$

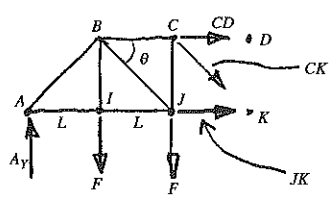
Solving: $A_x = 0$,

$A_y = 850$ kN

$H_y = 850$ kN

Note the symmetry:

Method of sections to find axial force in member JK.



$\theta = 45°$

$L = 8M$

$F = 340$ kN

$A_y = 850$ kN

$$\sum F_x: \quad CD + JK + CK\cos\theta = 0$$

$$\sum F_y: \quad A_y - 2F - CK\sin\theta = 0$$

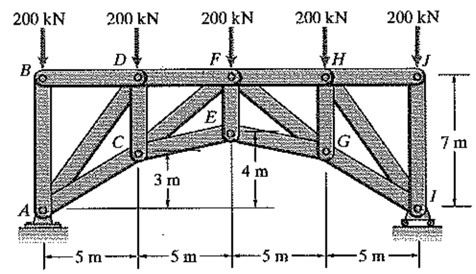
$$+\sum M_C: \quad L(JK) + L(F) - 2L(A_y) = 0$$

Solving, $JK = 1360$ kN (T)

Also, $CK = 240.4$ kN (T)

$CD = -1530$ kN (C)

Problem 6.50 For the bridge truss shown, use the method of sections to determine the axial forces in members *CE*, *CF*, and *DF*.

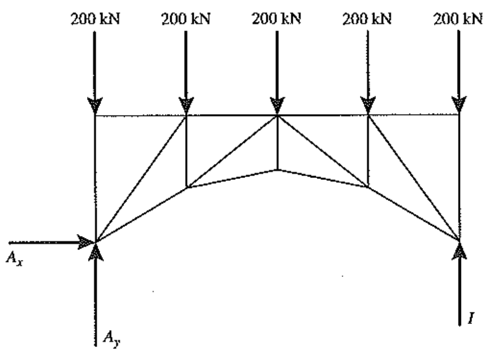


Solution: From the entire structure we find the reactions at A

$$\sum F_x : A_x = 0$$

$$\sum M_I : (200 \text{ kN})(5 \text{ m}) + (200 \text{ kN})(10 \text{ m}) + (200 \text{ kN})(15 \text{ m})$$

$$+ (200 \text{ kN})(20 \text{ m}) - A_y(20 \text{ m}) = 0 \Rightarrow A_y = 500 \text{ kN}$$



Now we cut through DF, CF, and CE and use the left section.

$$\sum M_C : (200 \text{ kN})(5 \text{ m}) - A_y(5 \text{ m}) + A_x(3 \text{ m}) - F_{DF}(4 \text{ m}) = 0$$

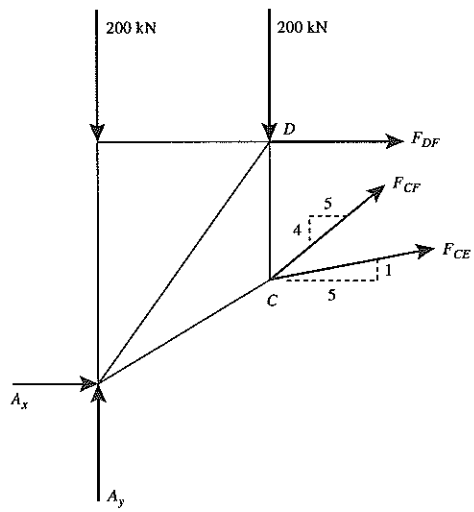
$$\Rightarrow F_{DF} = -375 \text{ kN}$$

$$\sum M_F : (200 \text{ kN})(10 \text{ m}) + (200 \text{ kN})(5 \text{ m}) - A_y(10 \text{ m}) + A_x(7 \text{ m})$$

$$+ \frac{5}{\sqrt{26}} F_{CE}(4 \text{ m}) - \frac{1}{\sqrt{26}} F_{CE}(5 \text{ m}) = 0 \Rightarrow F_{CE} = 680 \text{ kN}$$

$$\sum F_x : A_x + F_{DF} + \frac{5}{\sqrt{26}} F_{CE} + \frac{5}{\sqrt{41}} F_{CF} = 0$$

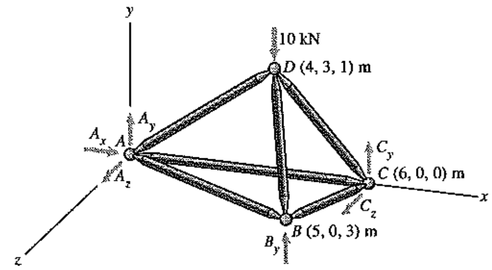
$$\Rightarrow F_{CF} = -374 \text{ kN}$$



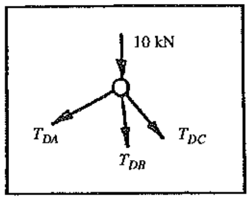
Summary:

$$F_{DF} = 375 \text{ kN}(C), \quad F_{CE} = 680 \text{ kN}(T), \quad F_{CF} = 374 \text{ kN}(C)$$

Problem 6.58 The space truss supports a vertical 10-kN load at D. The reactions at the supports at joints A, B, and C are shown. What are the axial forces in the members AD, BD, and CD?



Solution: Consider the joint D only. The position vectors parallel to the members from D are



$$\mathbf{r}_{DA} = -4\mathbf{i} - 3\mathbf{j} - \mathbf{k},$$

$$\mathbf{r}_{DB} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k},$$

$$\mathbf{r}_{DC} = 2\mathbf{i} - 3\mathbf{j} - \mathbf{k}.$$

The unit vectors parallel to the members from D are:

$$\mathbf{e}_{DA} = \frac{\mathbf{r}_{DA}}{|\mathbf{r}_{DA}|} = -0.7845\mathbf{i} - 0.5883\mathbf{j} - 0.1961\mathbf{k}$$

$$\mathbf{e}_{DB} = \frac{\mathbf{r}_{DB}}{|\mathbf{r}_{DB}|} = 0.2673\mathbf{i} - 0.8018\mathbf{j} + 0.5345\mathbf{k}$$

$$\mathbf{e}_{DC} = \frac{\mathbf{r}_{DC}}{|\mathbf{r}_{DC}|} = 0.5345\mathbf{i} - 0.8018\mathbf{j} - 0.2673\mathbf{k}$$

The equilibrium conditions for the joint D are

$$\sum \mathbf{F} = T_{DA}\mathbf{e}_{DA} + T_{DB}\mathbf{e}_{DB} + T_{DC}\mathbf{e}_{DC} - \mathbf{F}_D = 0,$$

from which

$$\sum F_x = -0.7845T_{DA} + 0.2673T_{DB} + 0.5345T_{DC} = 0$$

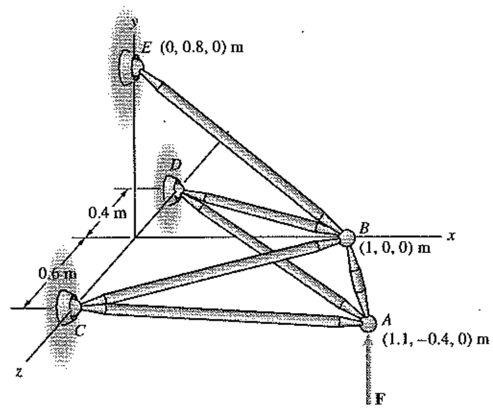
$$\sum F_y = -0.5883T_{DA} - 0.8018T_{DB} - 0.8108T_{DC} - 10 = 0$$

$$\sum F_z = -0.1961T_{DA} + 0.5345T_{DB} - 0.2673T_{DC} = 0.$$

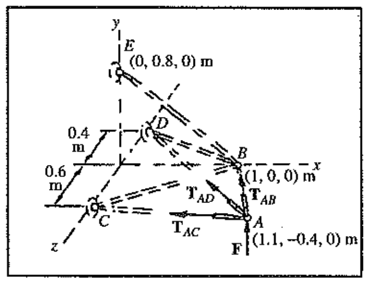
Solve: $\boxed{T_{DA} = -4.721 \text{ kN } (C)}$, $\boxed{T_{DB} = -4.157 \text{ kN } (C)}$

$\boxed{T_{DC} = -4.850 \text{ kN } (C)}$

Problem 6.63 The space truss shown models an airplane's landing gear. It has ball and socket supports at C, D, and E. If the force exerted at A by the wheel is $\mathbf{F} = 40\mathbf{j}$ (kN), what are the axial forces in members AB, AC, and AD?



Solution: The important points in this problem are A (1.1, −0.4, 0), B (1, 0, 0), C (0, 0, 6), and D (0, 0, −0.4). We do not need point E as all of the needed unknowns converge at A and none involve the location of point E. The unit vectors along AB, AC, and AD are



$$\mathbf{u}_{AB} = -0.243\mathbf{i} + 0.970\mathbf{j} + 0\mathbf{k},$$

$$\mathbf{u}_{AC} = -0.836\mathbf{i} + 0.304\mathbf{j} + 0.456\mathbf{k},$$

and $\mathbf{u}_{AD} = -0.889\mathbf{i} + 0.323\mathbf{j} - 0.323\mathbf{k}$.

The forces can be written as

$$\mathbf{T}_{RS} = T_{RS}\mathbf{u}_{RS} = T_{RSX}\mathbf{i} + T_{RSY}\mathbf{j} + T_{RSZ}\mathbf{k},$$

where **RS** takes on the values AB, AC, and AD. We now have three forces written in terms of unknown magnitudes and known directions. The equations of equilibrium for point A are

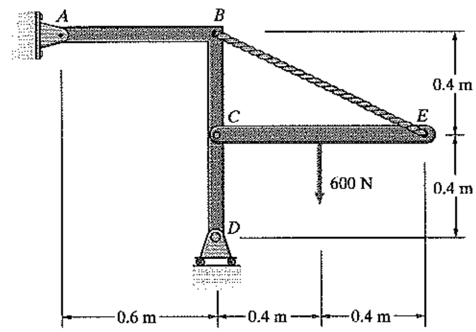
$$\sum F_x = T_{AB}u_{ABX} + T_{AC}u_{ACX} + T_{AD}u_{ADX} + F_X = 0,$$

$$\sum F_y = T_{AB}u_{ABY} + T_{AC}u_{ACY} + T_{AD}u_{ADY} + F_Y = 0,$$

and $\sum F_z = T_{AB}u_{ABZ} + T_{AC}u_{ACZ} + T_{AD}u_{ADZ} + F_Z = 0,$

where $\mathbf{F} = F_X\mathbf{i} + F_Y\mathbf{j} + F_Z\mathbf{k} = 40\mathbf{j}$ kN. Solving these equations for the three unknowns, we obtain $T_{AB} = -45.4$ kN (compression), $T_{AC} = 5.26$ kN (tension), and $T_{AD} = 7.42$ kN (tension).

Problem 6.76 Determine the reactions on member $ABCD$ at A, C, and D.

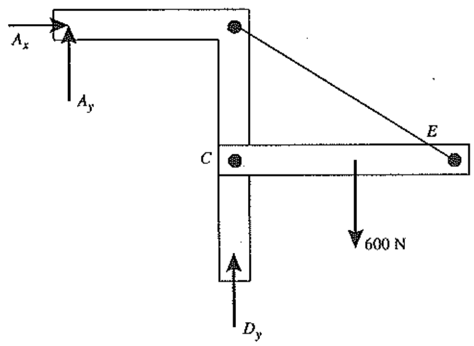


Solution: Consider the entire structure first

$\sum M_A : D_y(0.6 \text{ m}) - (600 \text{ N})(1.0 \text{ m}) = 0 \Rightarrow D_y = 1000 \text{ N}$

$\sum F_x : A_x = 0$

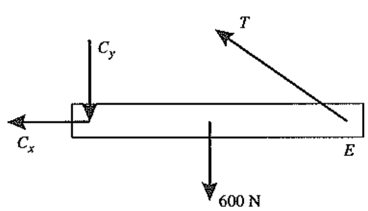
$\sum F_y : A_y + D_y - 600 \text{ N} = 0 \Rightarrow A_y = -400 \text{ N}$



Now examine bar CE. Note that the reactions on ABD are opposite to those on CE.

$\sum M_E : (600 \text{ N})(0.4 \text{ m}) + C_y(0.8 \text{ m}) = 0 \Rightarrow C_y = -300 \text{ N}$

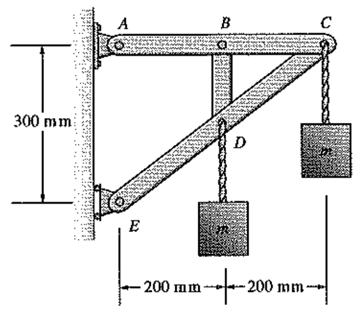
$\sum M_B : -C_x(0.4 \text{ m}) - (600 \text{ N})(0.4 \text{ m}) = 0 \Rightarrow C_x = -600 \text{ N}$



In Summary we have

$A_x = 0,\ A_y = -400 \text{ N}$
$C_x = -600 \text{ N},\ C_y = -300 \text{ N}$
$D_x = 0,\ D_y = 1000 \text{ N}$

Problem 6.80 The mass $m = 120$ kg. Determine the forces on member ABC, presenting your answers as shown in Fig. 6.25.



Solution: The equations of equilibrium for the entire frame are

$$\sum F_X = A_X + E_X = 0,$$

$$\sum F_Y = A_Y - 2mg = 0,$$

and summing moments at A,

$$\sum M_A = (0.3)E_X - (0.2)mg - (0.4)mg = 0.$$

Solving yields $A_X = -2354$ N, $A_Y = 2354$ N, and $E_X = 2354$ N.

Member ABC: The equilibrium equations are

$$\sum F_X = A_X + C_X = 0,$$

$$\sum F_Y = A_Y - B_Y + C_Y = 0,$$

and $\sum M_A = -(0.2)B_Y + (0.4)C_Y = 0.$

We have three equations in the three unknowns B_Y, C_X, and C_Y. Solving, we get $B_Y = 4708$ N, $C_X = 2354$ N, and $C_Y = 2354$ N. This gives all of the forces on member ABC. A similar analysis can be made for each of the other members in the frame. The results of solving for all of the forces in the frame is shown in the figure.