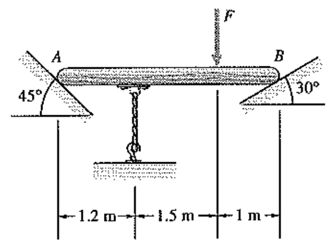
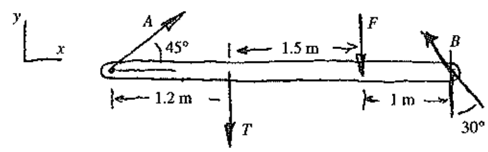
Problem 5.3 The beam is subjected to a load $F = 400$ N and is supported by the rope and the smooth surfaces at A and B.

(a) Draw the free-body diagram of the beam.
(b) What are the magnitudes of the reactions at A and B?



Solution:



$$\sum F_X = 0: \quad A\cos 45^\circ - B\sin 30^\circ = 0$$

$$\sum F_Y = 0: \quad A\sin 45^\circ + B\cos 30^\circ - T - 400\text{ N} = 0$$

$$\circlearrowleft + \sum M_A = 0: \quad -1.2T - 2.7(400) + 3.7B\cos 30^\circ = 0$$

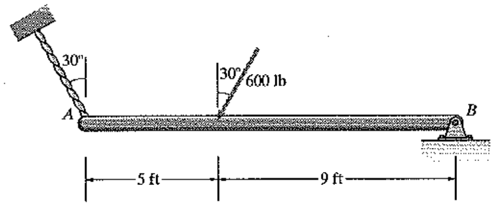
Solving, we get

$A = 271$ N

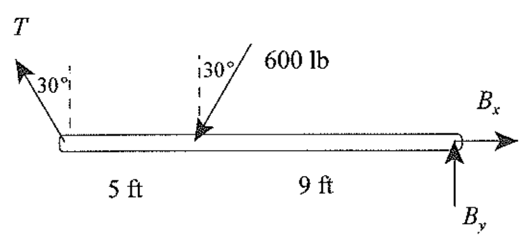
$B = 383$ N

$T = 124$ N

Problem 5.4 (a) Draw the free-body diagram of the beam. (b) Determine the tension in the rope and the reactions at B.



Solution: Let T be the tension in the rope.



The equilibrium equations are:

$\Sigma F_x : -T\sin 30^\circ - (600\text{ lb})\sin 30^\circ + B_x = 0$

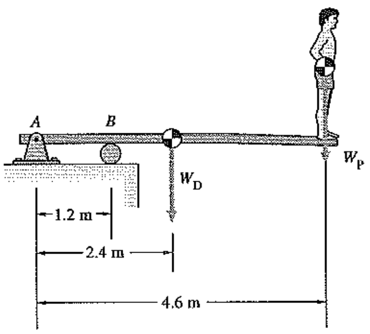
$\Sigma F_y : T\cos 30^\circ - (600\text{ lb})\cos 30^\circ + B_y = 0$

$\Sigma M_B : (600\text{ lb})\cos 30^\circ(9\text{ ft}) - T\cos 30^\circ(14\text{ ft}) = 0$

Solving yields $\boxed{T = 368\text{ lb}, B_x = 493\text{ lb}, B_y = 186\text{ lb}}$

Problem 5.6 The masses of the person and the diving board are 54 kg and 36 kg, respectively. Assume that they are in equilibrium.

(a) Draw the free-body diagram of the diving board.
(b) Determine the reactions at the supports A and B.



Solution:

(a)

(b) $\sum F_X = 0$: $A_X = 0$

$\sum F_Y = 0$: $A_Y + B_Y - (54)(9.81) - 36(9.81) = 0$

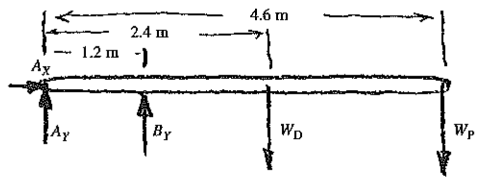
$\sum M_A = 0$: $1.2B_Y - (2.4)(36)(9.81)$

$- (4.6)(54)(9.81) = 0$

Solving: $A_X = 0$ N

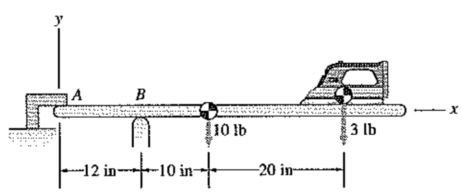
$A_Y = -1.85$ kN

$B_Y = 2.74$ kN



Problem 5.7 The ironing board has supports at A and B that can be modeled as roller supports.

(a) Draw the free-body diagram of the ironing board.
(b) Determine the reactions at A and B.



Solution: The system is in equilibrium.

(a) The free-body diagram is shown.
(b) The sums of the forces are:

$$\sum F_X = 0,$$

$$\sum F_Y = -F_A + F_B - 10 - 3 = 0.$$

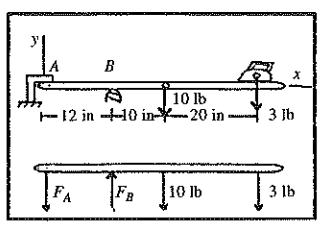
The sum of the moments about A is

$$\sum M_A = 12F_B - 22(10) - 42(3) = 0,$$

from which $F_B = \frac{346}{12} = 28.833$ in.

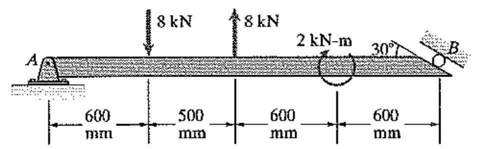
Substitute into the force balance equation:

$F_A = 13 - F_B = +15.833$ lb



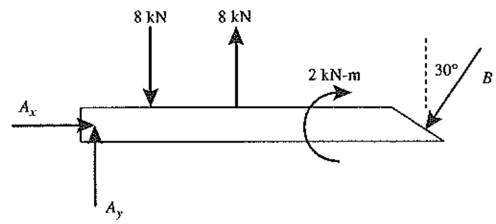
Problem 5.12 (a) Draw the free-body diagram of the beam.

(b) Determine the reactions at the pin support A.



Solution:

(a) The FBD



(b) The equilibrium equations

$$\sum M_A : -(8\text{ kN})(0.6\text{ m}) + (8\text{ kN})(1.1\text{ m}) - 2\text{ kNm}$$

$$- B\cos 30^\circ (2.3\text{ m}) = 0$$

$$\sum F_x : A_x - B\sin 30^\circ = 0$$

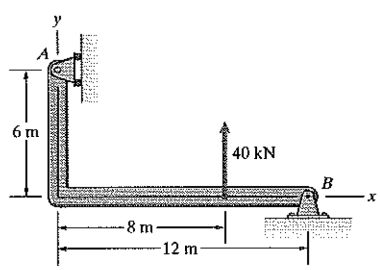
$$\sum F_y : A_y - 8\text{ kN} + 8\text{ kN} - B\cos 30^\circ = 0$$

Solving

$$\boxed{A_x = 0.502\text{ kN},\ A_y = 0.870\text{ kN},\ B = 1.004\text{ kN}}$$

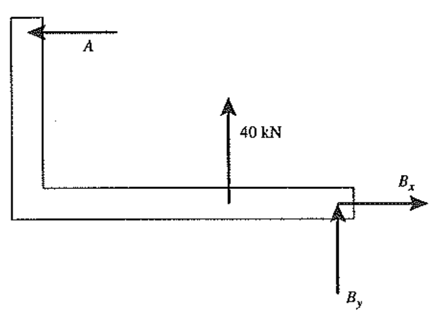
Problem 5.13 (a) Draw the free-body diagram of the beam.

(b) Determine the reactions at the supports.



Solution:

(a) The FBD



(b) The equilibrium equations

$$\sum M_B : -(40\text{ kN})(4\text{ m}) + A(6\text{ m}) = 0$$

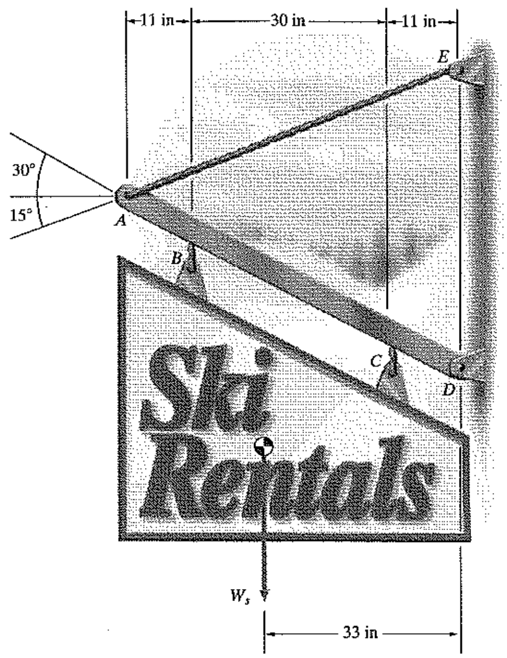
$$\sum F_x : -A + B_x = 0$$

$$\sum F_y : 40\text{ kN} + B_y = 0$$

Solving we find

$$\boxed{A = B_x = 26.7\text{ kN},\ B_y = -40\text{ kN}}$$

Problem 5.34 The sign's weight $W_S = 32$ lb acts at the point shown. The 10-lb weight of bar AD acts at the midpoint of the bar. Determine the tension in the cable AE and the reactions at D.



Solution: Treat the bar AD and sign as one single object. Let T_{AE} be the tension in the cable. The equilibium equations are

$$\Sigma F_x : T_{AE} \cos 15^\circ + D_x = 0$$

$$\Sigma F_y : T_{AE} \sin 15^\circ + D_y - W_s - (10 \text{ lb}) = 0$$

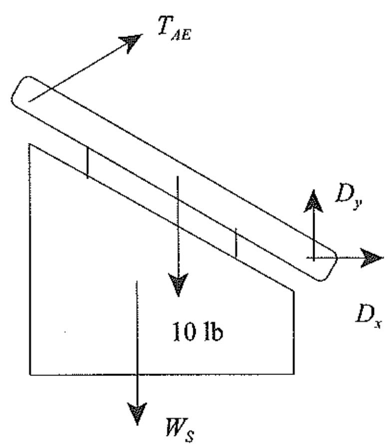
$$\Sigma M_D : -T_{AE} \cos 15^\circ (52 \text{ in}) \tan 30^\circ$$

$$- T_{AE} \sin 15^\circ (52 \text{ in})$$

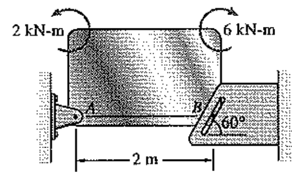
$$+ (32 \text{ lb})(33 \text{ in}) + (10 \text{ lb})(26 \text{ in}) = 0$$

Solving yields

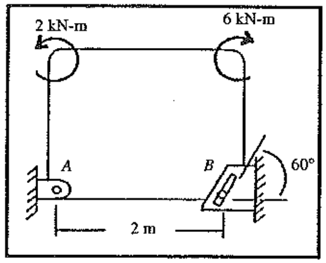
$$T_{AE} = 31.0 \text{ lb}, \quad D_x = -29.9 \text{ lb}, \quad D_y = 34.0 \text{ lb}$$



Problem 5.42 The plate is supported by a pin in a smooth slot at B. What are the reactions at the supports?



Solution: The pinned support is a two force reaction support. The smooth pin is a roller support, with a one force reaction. The reaction at B forms an angle of $90° + 60° = 150°$ with the positive x axis. The sum of the forces:



$$\sum F_X = A_X + B\cos 150° = 0$$

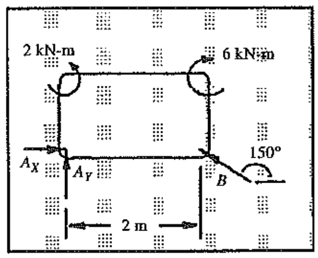
$$\sum F_Y = A_Y + B\sin 150° = 0$$

The sum of the moments about B is

$$\sum M_B = -2A_Y + 2 - 6 = 0,$$

from which

$$A_Y = -\frac{4}{2} = -2 \text{ kN}.$$



Substitute into the force equations to obtain

$$B = \frac{A_Y}{\sin 150°} = 4 \text{ kN},$$

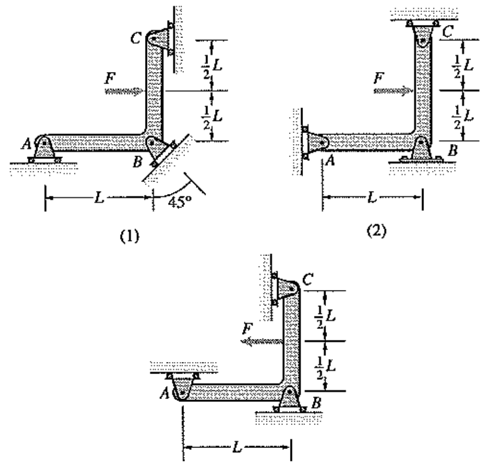
and $A_X = -B\cos 150° = 3.464$ kN.

The horizontal and vertical reactions at B are

$$B_X = 4\cos 150° = -3.464 \text{ kN},$$

and $B_Y = 4\sin 150° = 2$ kN.

Problem 5.76 State whether each of the L-shaped bars shown is properly or improperly supported. If a bar is properly supported, determine the reactions at its supports. (See Active Example 5.6.)



Solution:

(1) *is improperly constrained.* The reactions intersect at a point P, and the force exerts a moment about that point.

(2) *is improperly constrained.* The reactions intersect at a point P and the force exerts a moment about that point.

(3) *is properly constrained.* The sum of the forces:

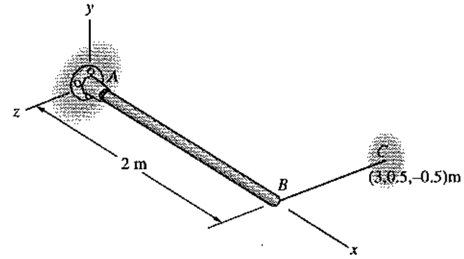
$$\sum F_X = C - F = 0,$$

from which $C = F$.

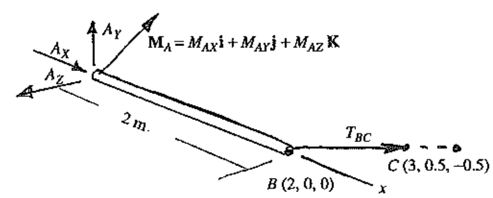
$$\sum F_Y = -A + B = 0,$$

from which $A = B$. The sum of the moments about B: $LA + \frac{L}{2}F - LC = 0$, from which $A = \frac{1}{2}F$, and $B = \frac{1}{2}F$

Problem 5.78 The bar AB has a built-in support at A. The tension in cable BC is 8 kN. Determine the reactions at A.



Solution:



$\mathbf{M}_A = M_{Ax}\mathbf{i} + M_{Ay}\mathbf{j} + M_{Az}\mathbf{k}$

We need the unit vector $\mathbf{e}_{BC}$

$$\mathbf{e}_{BC} = \frac{(x_C - x_B)\mathbf{i} + (y_C - y_B)\mathbf{j} + (z_C - z_B)\mathbf{k}}{\sqrt{(x_C - x_B)^2 + (y_C - y_B)^2 + (z_C - z_B)^2}}$$

$\mathbf{e}_{BC} = 0.816\mathbf{i} + 0.408\mathbf{j} - 0.408\mathbf{k}$

$\mathbf{T}_{BC} = (8 \text{ kN})\mathbf{e}_{BC}$

$\mathbf{T}_{BC} = 6.53\mathbf{i} + 3.27\mathbf{j} - 3.27\mathbf{k}$ (kN)

The moment of $\mathbf{T}_{BC}$ about A is

$$\mathbf{M}_{BC} = \mathbf{r}_{AB} \times T_{BC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 0 \\ 6.53 & 3.27 & -3.27 \end{vmatrix}$$

$\mathbf{M}_{BC} = \mathbf{r}_{AB} \times \mathbf{T}_{BC} = 0\mathbf{i} + 6.53\mathbf{j} + 6.53\mathbf{k}$ (kN-m)

Equilibrium Eqns.

$\sum F_X$: $A_X + T_{BC_X} = 0$

$\sum F_Y$: $A_Y + T_{BC_Y} = 0$

$\sum F_Z$: $A_Z + T_{BC_Z} = 0$

$\sum M_X$: $M_{AX} + M_{BC_X} = 0$

$\sum M_Y$: $M_{AY} + M_{BC_Y} = 0$

$\sum M_Z$: $M_{AZ} + M_{BC_Z} = 0$

Solving, we get

$A_X = -6.53$ (kN),

$A_Y = -3.27$ (kN),

$A_Z = 3.27$ (kN)

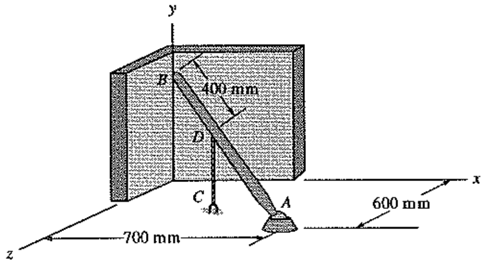
$M_{Ax} = 0$,

$M_{Ay} = -6.53$ (kN-m),

$M_{Az} = -6.53$ (kN-m)

Problem 5.98 The 1.1-m bar is supported by a ball and socket support at A and the two smooth walls. The tension in the vertical cable CD is 1 kN.

(a) Draw the free-body diagram of the bar.
(b) Determine the reactions at A and B.



Solution:

(a) The ball and socket cannot support a couple reaction, but can support a three force reaction. The smooth surface supports one-force normal to the surface. The cable supports one force parallel to the cable.

(b) The strategy is to determine the moments about A, which will contain only the unknown reaction at B. This will require the position vectors of B and D relative to A, which in turn will require the unit vector parallel to the rod. The angle formed by the bar with the horizontal is required to determine the coordinates of B:

$$\alpha = \cos^{-1}\left(\frac{\sqrt{0.6^2 + 0.7^2}}{1.1}\right) = 33.1^\circ.$$

The coordinates of the points are: A (0.7, 0, 0.6), B (0, 1.1 (sin 33.1°), 0) = (0, 0.6, 0), from which the vector parallel to the bar is

$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A = -0.7\mathbf{i} + 0.6\mathbf{j} - 0.6\mathbf{k} \text{ (m)}.$$

The unit vector parallel to the bar is

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{1.1} = -0.6364\mathbf{i} + 0.5455\mathbf{j} - 0.5455\mathbf{k}.$$

The vector location of the point D relative to A is

$$\mathbf{r}_{AD} = (1.1 - 0.4)\mathbf{e}_{AB} = 0.7\mathbf{e}_{AB}$$

$$= -0.4455\mathbf{i} + 0.3819\mathbf{j} - 0.3819\mathbf{k}.$$

The reaction at B is horizontal, with unknown x-component and z-components. The sum of the moments about A is

$$\sum \mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{B} + \mathbf{r}_{AD} \times \mathbf{D} = 0 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.7 & 0.6 & -0.6 \\ B_X & 0 & B_Z \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.4455 & 0.3819 & -0.3819 \\ 0 & -1 & 0 \end{vmatrix} = 0$$

Expand and collect like terms:

$$\sum \mathbf{M}_A = (0.6B_Z - 0.3819)\mathbf{i} - (0.6B_X - 0.7B_Z)\mathbf{j}$$

$$+(-0.6B_X + 0.4455)\mathbf{k} = 0.$$

From which,

$$B_Z = \frac{0.3819}{0.6} = 0.6365 \text{ kN},$$

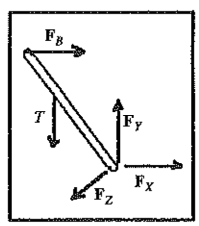
$$B_X = \frac{0.4455}{0.6} = 0.7425 \text{ kN}.$$

The reactions at A are determined from the sums of the forces:

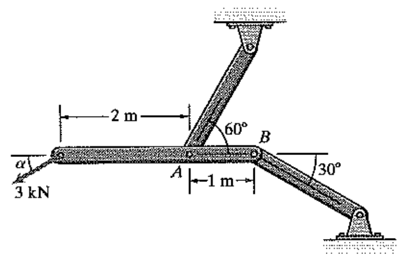
$\sum \mathbf{F}_X = (B_X + A_X)\mathbf{i} = 0$, from which $A_X = -0.7425$ kN.

$\sum \mathbf{F}_Y = (A_Y - 1)\mathbf{j} = 0$, from which $A_Y = 1$ kN.

$\sum \mathbf{F}_Z = (B_Z + A_Z)\mathbf{k} = 0$, from which $A_Z = -0.6365$ kN



Problem 5.126 Use the fact that the horizontal bar is a three-force member to determine the angle α and the magnitudes of the reactions at A and B. Assume that $0 \le \alpha \le 90°$.



Solution: The forces at A and B are parallel to the respective bars since these bars are 2-force members. Since the horizontal bar is a 3-force member, all of the forces must intersect at a point. Thus we have the following picture:

From geometry we see that

$d = 1 \text{ m} \cos 30°$

$d \sin 30° = e \sin \alpha$

$d \cos 30° + e \cos \alpha = 3 \text{ m}$

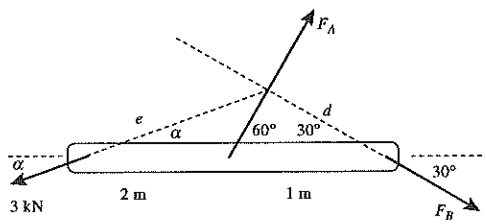
Solving we find

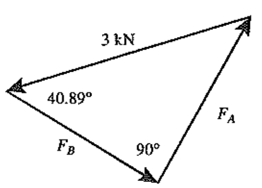
$\boxed{\alpha = 10.89°}$

To find the other forces we look at the force triangle

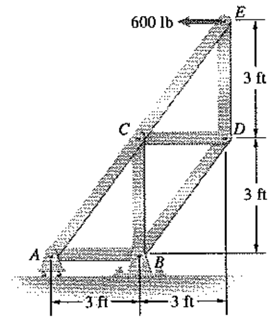
$F_B = 3 \text{ kN} \cos 40.89° = 2.27 \text{ kN}$

$F_A = 3 \text{ kN} \sin 40.89° = 1.964 \text{ kN}$





Problem 6.4 Determine the axial forces in members BC and CD of the truss.



Solution: The free-body diagrams for joints E, D, and C are shown. The angle α is

$$\alpha = \tan^{-1}(3/4) = 36.9^\circ$$

Using Joint E, we have

$\Sigma F_x : -(600 \text{ lb}) - T_{CE} \sin\alpha = 0$

$\Sigma F_y : -T_{CE} \cos\alpha - T_{DE} = 0$

Using Joint D, we have

$\Sigma F_x : -T_{CD} - T_{BD} \sin\alpha = 0$

$\Sigma F_y : T_{DE} - T_{BD} \cos\alpha = 0$

Finally, using Joint C, we have

$\Sigma F_x : T_{CD} + T_{CD} \sin\alpha - T_{AC} \sin\alpha = 0$

$\Sigma F_y : T_{CE} \cos\alpha - T_{AC} \cos\alpha - T_{BC} = 0$

Solving these six equations yields

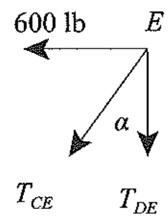
$T_{CE} = -1000 \text{ lb}, T_{DE} = 800 \text{ lb}$

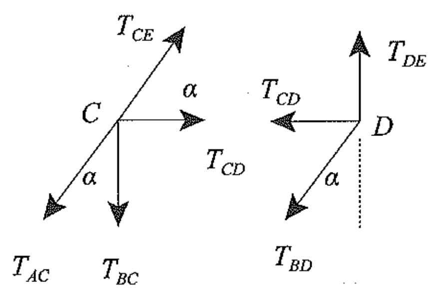
$T_{CD} = -600 \text{ lb}, T_{AC} = -2000 \text{ lb}$

$T_{BC} = 800 \text{ lb}, T_{BD} = 1000 \text{ lb}$

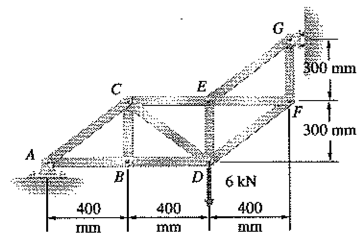
A positive value means tension and a negative value means compression

Thus $\boxed{BC : 800 \text{ lb (T)}, \quad CD : 600 \text{ lb (C)}}$





Problem 6.10 Determine the axial forces in members *BD*, *CD*, and *CE* of the truss.



Solution: The free-body diagrams of the entire truss and of joints *A*, *B*, and *C* are shown. The angle

$$\alpha = \tan^{-1}(3/4) = 36.9°$$

From the free-body diagram of the entire truss

$\Sigma F_y : A_y - 6 \text{ kN} = 0$

$\Sigma M_G : (6 \text{ kN})(400 \text{ mm}) + A_x(600 \text{ mm})$

$- A_y(1200 \text{ mm}) = 0$

Solving, $A_x = 8$ kN, $A_y = 6$ kN

Using joint *A*,

$\Sigma F_x : A_x + T_{AB} + T_{AC} \cos\alpha = 0$

$\Sigma F_y : A_y + T_{AC} \sin\alpha = 0$

Solving we find

$$T_{AB} = 0, \quad T_{AC} = -10 \text{ kN}$$

Because joint *B* consists of three members, two of which are parallel, and is subjected to no external load, we can recognize that

$$T_{BD} = T_{AB} = 0 \quad \text{and} \quad T_{BD} = 0$$

Finally we examine joint *C*

$$\left.\begin{array}{l}\Sigma F_x : T_{CE} + T_{CD}\cos\alpha - T_{AC}\cos\alpha = 0 \\ \Sigma F_y : -T_{AC}\sin\alpha - T_{CD}\sin\alpha - T_{BC} = 0\end{array}\right\} \Rightarrow T_{CD} = 10 \text{ kN}, T_{CE} = -16 \text{ kN}$$

In summary **BD : 0, CD : 10 kN (T), CE : 16 kN (C)**