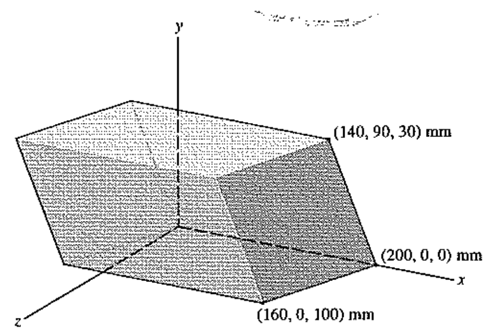
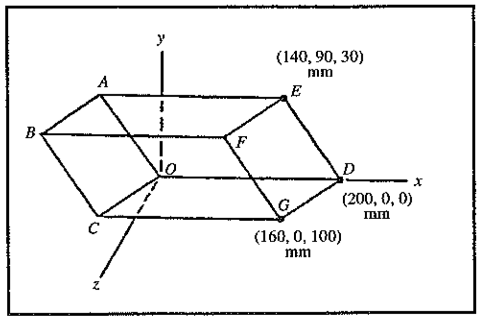
Problem 2.144 Use the mixed triple product to calculate the volume of the parallelepiped.



Solution: We are given the coordinates of point D. From the geometry, we need to locate points A and C. The key to doing this is to note that the length of side OD is 200 mm and that side OD is the x axis. Sides OD, AE, and CG are parallel to the x axis and the coordinates of the point pairs (O and D), (A and E), and (C and D) differ only by 200 mm in the x coordinate. Thus, the coordinates of point A are (−60, 90, 30) mm and the coordinates of point C are (−40, 0, 100) mm. Thus, the vectors $\mathbf{r}_{OA}$, $\mathbf{r}_{OD}$, and $\mathbf{r}_{OC}$ are $\mathbf{r}_{OD} = 200\mathbf{i}$ mm, $\mathbf{r}_{OA} = -60\mathbf{i} + 90\mathbf{j} + 30\mathbf{k}$ mm, and $\mathbf{r}_{OC} = -40\mathbf{i} + 0\mathbf{j} + 100\mathbf{k}$ mm. The mixed triple product of the three vectors is the volume of the parallelepiped. The volume is



$$\mathbf{r}_{OA} \cdot (\mathbf{r}_{OC} \times \mathbf{r}_{OD}) = \begin{vmatrix} -60 & 90 & 30 \\ -40 & 0 & 100 \\ 200 & 0 & 0 \end{vmatrix}$$

$$= -60(0) + 90(200)(100) + (30)(0) \text{ mm}^3$$

$$= 1{,}800{,}000 \text{ mm}^3$$

Problem 2.145 By using Eqs. (2.23) and (2.34), show that

$$\mathbf{U} \cdot (\mathbf{V} \times \mathbf{W}) = \begin{vmatrix} U_x & U_y & U_z \\ V_x & V_y & V_z \\ W_x & W_y & W_z \end{vmatrix}$$

Solution: One strategy is to expand the determinant in terms of its components, take the dot product, and then collapse the expansion. Eq. (2.23) is an expansion of the dot product: Eq. (2.23): $\mathbf{U} \cdot \mathbf{V} = U_X V_X + U_Y V_Y + U_Z V_Z$. Eq. (2.34) is the determinant representation of the cross product:

$$\text{Eq. (2.34) } \mathbf{U} \times V = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ U_X & U_Y & U_Z \\ V_X & V_Y & V_Z \end{vmatrix}$$

For notational convenience, write $\mathbf{P} = (\mathbf{U} \times \mathbf{V})$. Expand the determinant about its first row:

$$\mathbf{P} = \mathbf{i}\begin{vmatrix} U_Y & U_Z \\ V_Y & V_Z \end{vmatrix} - \mathbf{j}\begin{vmatrix} U_X & U_Z \\ V_X & V_Z \end{vmatrix} + \mathbf{k}\begin{vmatrix} U_X & U_Z \\ V_X & V_Z \end{vmatrix}$$

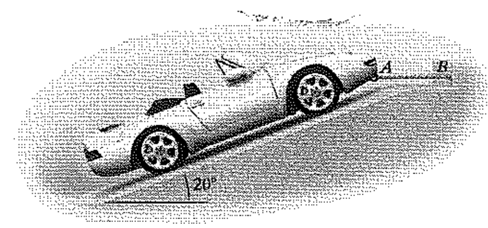
Since the two-by-two determinants are scalars, this can be written in the form: $\mathbf{P} = \mathbf{i}P_X + \mathbf{j}P_Y + \mathbf{k}P_Z$ where the scalars P_X, P_Y, and P_Z are the two-by-two determinants. Apply Eq. (2.23) to the dot product of a vector $\mathbf{Q}$ with $\mathbf{P}$. Thus $\mathbf{Q} \cdot \mathbf{P} = Q_X P_X + Q_Y P_Y + Q_Z P_Z$. Substitute P_X, P_Y, and P_Z into this dot product

$$\mathbf{Q} \cdot \mathbf{P} = Q_X\begin{vmatrix} U_Y & U_Z \\ V_Y & V_Z \end{vmatrix} - Q_Y\begin{vmatrix} U_X & U_Z \\ V_X & V_Z \end{vmatrix} + Q_z\begin{vmatrix} U_X & U_Z \\ V_X & V_Z \end{vmatrix}$$

But this expression can be collapsed into a three-by-three determinant directly, thus:

$$\mathbf{Q} \cdot (\mathbf{U} \times \mathbf{V}) = \begin{vmatrix} Q_X & Q_Y & Q_Z \\ U_X & U_Y & U_Z \\ V_X & V_Y & V_Z \end{vmatrix}.$$ This completes the demonstration.

Problem 3.1 In Active Example 3.1, suppose that the angle between the ramp supporting the car is increased from 20° to 30°. Draw the free-body diagram of the car showing the new geometry. Suppose that the cable from A to B must exert a 1900-lb horizontal force on the car to hold it in place. Determine the car's weight in pounds.



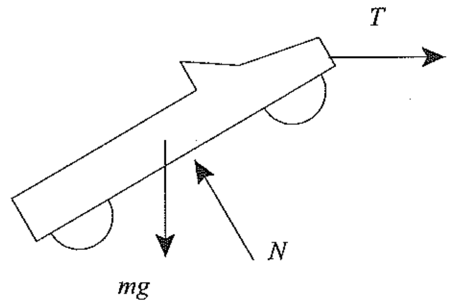
Solution: The free-body diagram is shown to the right. Applying the equilibrium equations

$\sum F_x : T - N \sin 30^\circ = 0,$

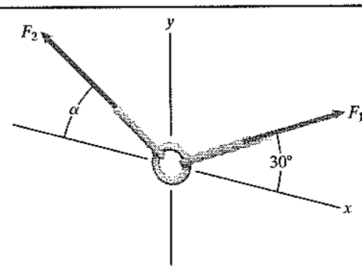
$\sum F_y : N \cos 30^\circ - mg = 0$

Setting $T = 1900$ lb and solving yields

$$N = 3800 \text{ lb}, \quad \boxed{mg = 3290 \text{ lb}}$$



Problem 3.2 The ring weighs 5 lb and is in equilibrium. The force $F_1 = 4.5$ lb. Determine the force F_2 and the angle α.



Solution: The free-body diagram is shown below the drawing. The equilibrium equations are

$\sum F_x : F_1 \cos 30^\circ - F_2 \cos\alpha = 0$

$\sum F_y : F_1 \sin 30^\circ + F_2 \sin\alpha - 5 \text{ lb} = 0$

We can write these equations as

$F_2 \sin\alpha = 5 \text{ lb} - F_1 \sin 30^\circ$

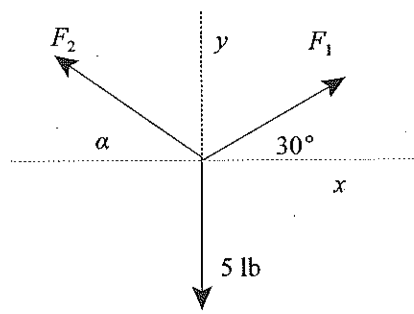
$F_2 \cos\alpha = F_1 \cos 30^\circ$

Dividing these equations and using the known value for F_1 we have.

$$\tan\alpha = \frac{5 \text{ lb} - (4.5 \text{ lb}) \sin 30^\circ}{(4.5 \text{ lb}) \cos 30^\circ} = 0.706 \Rightarrow \alpha = 35.2^\circ$$

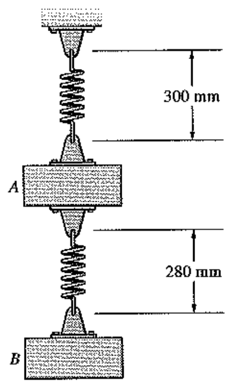
$$F_2 = \frac{(4.5 \text{ lb}) \cos 30^\circ}{\cos\alpha} = 4.77 \text{ lb}$$

$$\boxed{F_2 = 4.77 \text{ lb}, \alpha = 35.2^\circ}$$

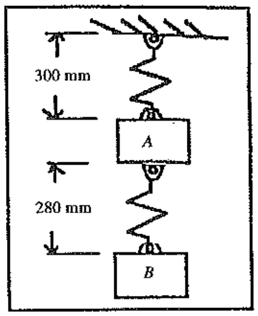


Problem 3.7 The two springs are identical, with unstretched lengths 250 mm and spring constants $k = 1200$ N/m.

(a) Draw the free-body diagram of block A.
(b) Draw the free-body diagram of block B.
(c) What are the masses of the two blocks?



Solution: The tension in the upper spring acts on block A in the positive Y direction, Solve the spring force-deflection equation for the tension in the upper spring. Apply the equilibrium conditions to block A. Repeat the steps for block B.

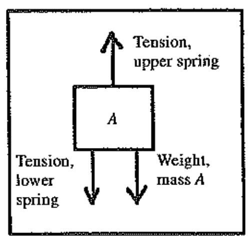


$$\mathbf{T}_{UA} = 0\mathbf{i} + \left(1200\ \frac{\text{N}}{\text{m}}\right)(0.3\ \text{m} - 0.25\ \text{m})\mathbf{j} = 0\mathbf{i} + 60\mathbf{j}\ \text{N}$$

Similarly, the tension in the lower spring acts on block A in the negative Y direction

$$\mathbf{T}_{LA} = 0\mathbf{i} - \left(1200\ \frac{\text{N}}{\text{m}}\right)(0.28\ \text{m} - 0.25\ \text{m})\mathbf{j} = 0\mathbf{i} - 36\mathbf{j}\ \text{N}$$

The weight is $\mathbf{W}_A = 0\mathbf{i} - |\mathbf{W}_A|\mathbf{j}$



The equilibrium conditions are

$$\sum \mathbf{F} = \sum \mathbf{F}_x + \sum \mathbf{F}_y = 0, \quad \sum \mathbf{F} = \mathbf{W}_A + \mathbf{T}_{UA} + \mathbf{T}_{LA} = 0$$

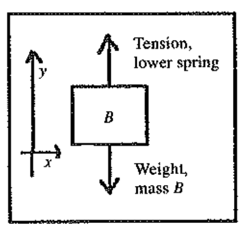
Collect and combine like terms in **i**, **j**

$$\sum \mathbf{F}_y = (-|\mathbf{W}_A| + 60 - 36)\mathbf{j} = 0$$

Solve $|\mathbf{W}_A| = (60 - 36) = 24$ N

The mass of A is

$$m_A = \frac{|\mathbf{W}_L|}{|\mathbf{g}|} = \frac{24\ \text{N}}{9.81\ \text{m/s}^2} = 2.45\ \text{kg}$$



The free body diagram for block **B** is shown.

The tension in the lower spring $\mathbf{T}_{LB} = 0\mathbf{i} + 36\mathbf{j}$

The weight: $\mathbf{W}_B = 0\mathbf{i} - |\mathbf{W}_B|\mathbf{j}$
Apply the equilibrium conditions to block B.

$$\sum \mathbf{F} = \mathbf{W}_B + \mathbf{T}_{LB} = 0$$

Collect and combine like terms in **i**, **j**:

$$\sum \mathbf{F}_y = (-|\mathbf{W}_B| + 36)\mathbf{j} = 0$$

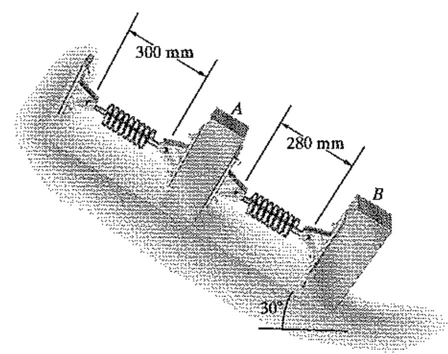
Solve: $|\mathbf{W}_B| = 36$ N

The mass of B is given by $m_B = \dfrac{|\mathbf{W}_B|}{|\mathbf{g}|} = \dfrac{36\ \text{N}}{9.81\ \text{m/s}^2} = 3.67$ kg

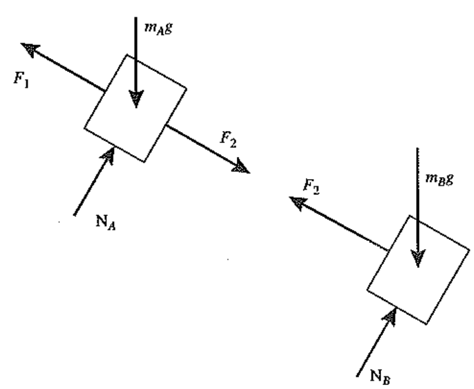
Problem 3.8 The two springs in Problem 3.7 are identical, with unstretched lengths of 250 mm. Suppose that their spring constant k is unknown and the sum of the masses of blocks A and B is 10 kg. Determine the value of k and the masses of the two blocks.

Solution: All of the forces are in the vertical direction so we will use scalar equations. First, consider the upper spring supporting both masses (10 kg total mass). The equation of equilibrium for block the entire assembly supported by the upper spring is A is $T_{UA} - (m_A + m_B)g = 0$, where $T_{UA} = k(\ell_U - 0.25)$ N. The equation of equilibrium for block B is $T_{UB} - m_B g = 0$, where $T_{UB} = k(\ell_L - 0.25)$ N. The equation of equilibrium for block A alone is $T_{UA} + T_{LA} - m_A g = 0$ where $T_{LA} = -T_{UB}$. Using $g = 9.81$ m/s^2, and solving simultaneously, we get $\boxed{k = 1962 \text{ N/m}, \; m_A = 4 \text{ kg, and } m_B = 6 \text{ kg}}$.

Problem 3.9 The inclined surface is smooth (Remember that "smooth" means that friction is negligble). The two springs are identical, with unstretched lengths of 250 mm and spring constants $k = 1200$ N/m. What are the masses of blocks A and B?



Solution:



$F_1 = (1200 \text{ N/m})(0.3 - 0.25)m = 60$ N

$F_2 = (1200 \text{ N/m})(0.28 - 0.25)m = 36$ N

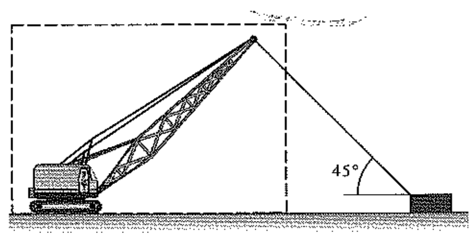
$\sum F_B \searrow: -F_2 + m_B g \sin 30^\circ = 0$

$\sum F_A \searrow: -F_1 + F_2 + m_A g \sin 30^\circ = 0$

Solving: $\boxed{m_A = 4.89 \text{ kg}, \; m_B = 7.34 \text{ kg}}$

Problem 3.10 The mass of the crane is 20,000 kg. The crane's cable is attached to a caisson whose mass is 400 kg. The tension in the cable is 1 kN.

(a) Determine the magnitudes of the normal and friction forces exerted on the crane by the level ground.
(b) Determine the magnitudes of the normal and friction forces exerted on the caisson by the level ground.



Strategy: To do part (a), draw the free-body diagram of the crane and the part of its cable within the dashed line.

Solution:

(a) $\sum F_y : N_{crane} - 196.2\text{ kN} - 1\text{ kN}\sin 45^\circ = 0$

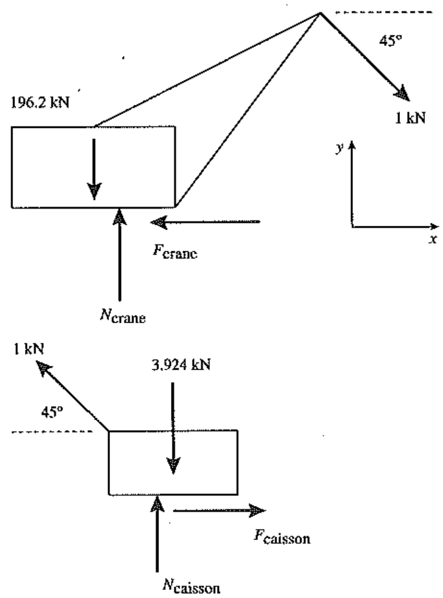
$\sum F_x : -F_{crane} + 1\text{ kN}\cos 45^\circ = 0$

$\boxed{N_{crane} = 196.9\text{ kN},\ F_{crane} = 0.707\text{ kN}}$

(b) $\sum F_y : N_{caisson} - 3.924\text{ kN} + 1\text{ kN}\sin 45^\circ = 0$

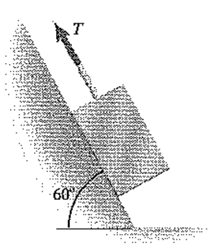
$\sum F_x : -1\text{ kN}\cos 45^\circ + F_{caisson} = 0$

$\boxed{N_{caisson} = 3.22\text{ kN},\ F_{caisson} = 0.707\text{ kN}}$



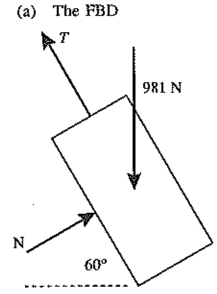
Problem 3.11 The inclined surface is smooth. The 100-kg crate is held stationary by a force T applied to the cable.

(a) Draw the free-body diagram of the crate.
(b) Determine the force T.



Solution:

(a) The FBD



(b) $\sum F \nwarrow : T - 981\text{ N}\sin 60^\circ = 0$

$\boxed{T = 850\text{ N}}$

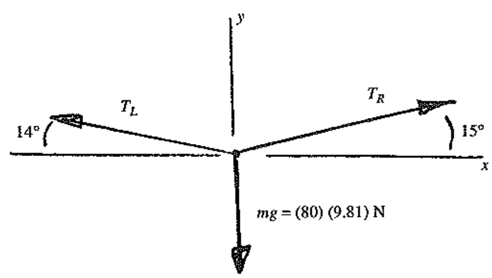
Problem 3.21 If the mass of the climber shown in Problem 3.20 is 80 kg, what are the tensions in the rope on the left and right sides?

Solution:

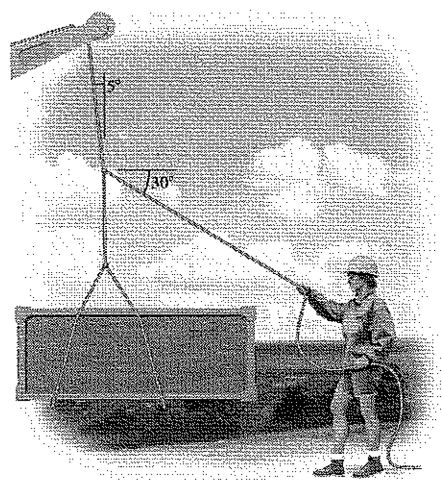
$$\begin{cases} \sum F_x = T_R\cos(15^\circ) - T_L\cos(14^\circ) = 0 \\ \sum F_y = T_R\sin(15^\circ) + T_L\sin(14^\circ) - mg = 0 \end{cases}$$

Solving, we get

$T_L = 1.56$ kN, $T_R = 1.57$ kN



Problem 3.22 The construction worker exerts a 20-lb force on the rope to hold the crate in equilibrium in the position shown. What is the weight of the crate?

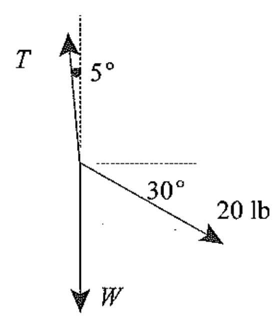


Solution: The free-body diagram is shown.
The equilibrium equations for the part of the rope system where the three ropes are joined are

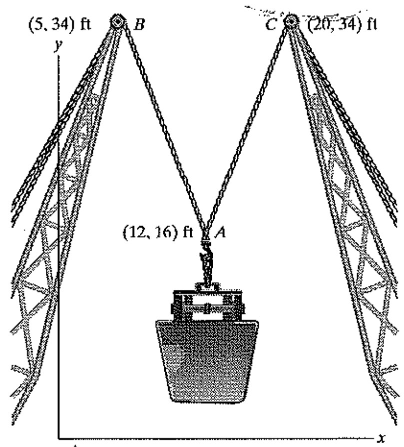
$$\sum F_x : (20 \text{ lb})\cos 30^\circ - T\sin 5^\circ = 0$$

$$\sum F_y : -(20 \text{ lb})\sin 30^\circ + T\cos 5^\circ - W = 0$$

Solving yields $\boxed{W = 188 \text{ lb}}$



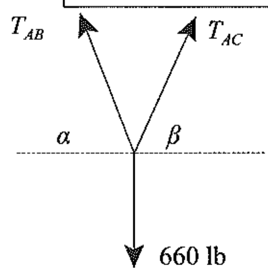
Problem 3.31 The bucket contains concrete and weighs 5800 lb. What are the tensions in the cables AB and AC?



Solution: The angles are

$$\alpha = \tan^{-1}\left(\frac{34-16}{12-5}\right) = 68.7^\circ$$

$$\beta = \tan^{-1}\left(\frac{34-16}{20-12}\right) = 66.0^\circ$$



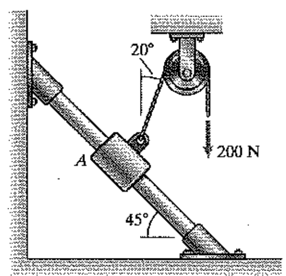
Now from equilibrium we have

$$\sum F_x : -T_{AB}\cos\alpha + T_{AC}\cos\beta = 0$$

$$\sum F_y : T_{AB}\sin\alpha + T_{AC}\sin\beta - 660\text{ lb} = 0$$

Solving yields $\boxed{T_{AB} = 319\text{ lb},\ T_{AC} = 470\text{ lb}}$

Problem 3.32 The slider A is in equilibrium and the bar is smooth. What is the mass of the slider?



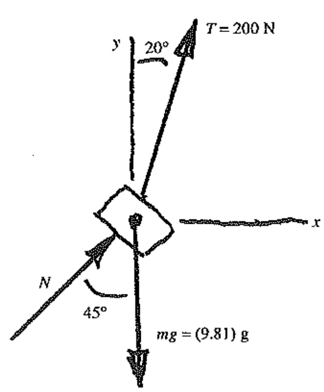
Solution: The pulley does not change the tension in the rope that passes over it. There is no friction between the slider and the bar.

Eqns. of Equilibrium:

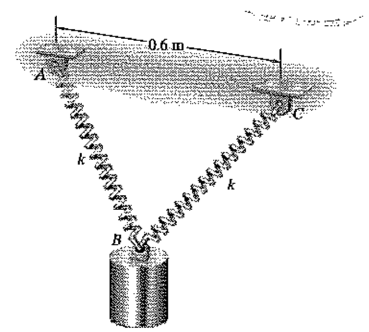
$$\begin{cases} \sum F_x = T\sin 20^\circ + N\cos 45^\circ = 0 \quad (T = 200\text{ N}) \\ \sum F_y = N\sin 45^\circ + T\cos 20^\circ - mg = 0 \quad g = 9.81\text{ m/s}^2 \end{cases}$$

Substituting for T and g, we have two eqns in two unknowns (N and m).

Solving, we get $N = -96.7$ N, $m = 12.2$ kg.



Problem 3.50 The two springs are identical, with unstretched length 0.4 m. When the 50-kg mass is suspended at B, the length of each spring increases to 0.6 m. What is the spring constant k?

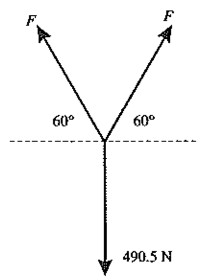


Solution:

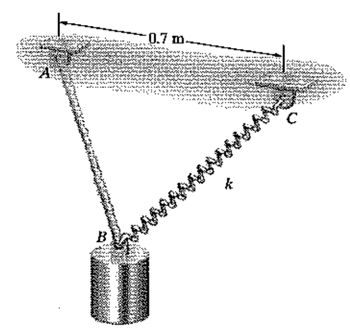
$F = k(0.6\text{ m} - 0.4\text{ m})$

$\sum F_y : 2F\sin 60^\circ - 490.5\text{ N} = 0$

$k = 1416\text{ N/m}$



Problem 3.51 The cable AB is 0.5 m in length. The *unstretched* length of the spring is 0.4 m. When the 50-kg mass is suspended at B, the length of the spring increases to 0.45 m. What is the spring constant k?



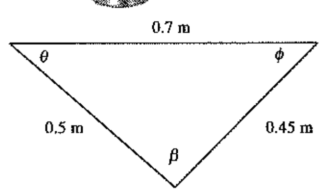
Solution: The Geometry

Law of Cosines and Law of Sines

$0.7^2 = 0.5^2 + 0.45^2 - 2(0.5)(0.45)\cos\beta$

$\dfrac{\sin\theta}{0.45\text{ m}} = \dfrac{\sin\phi}{0.5\text{ m}} = \dfrac{\sin\beta}{0.7\text{ m}}$

$\beta = 94.8^\circ,\ \theta = 39.8^\circ\ \phi = 45.4^\circ$



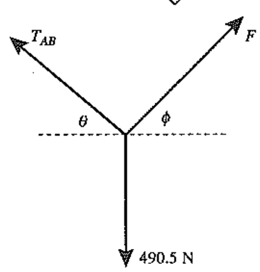
Now do the statics

$F = k(0.45\text{ m} - 0.4\text{ m})$

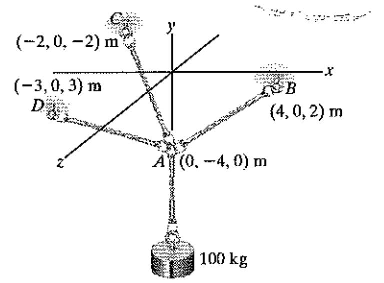
$\sum F_x : -T_{AB}\cos\theta + F\cos\phi = 0$

$\sum F_y : T_{AB}\sin\theta + F\sin\phi - 490.5\text{ N} = 0$

Solving: $k = 7560\text{ N/m}$



Problem 3.63 In Active Example 3.5, suppose that the attachment point B is moved to the point (5,0,0) m. What are the tensions in cables AB, AC, and AD?



Solution: The position vector from point A to point B can be used to write the force $\mathbf{T}_{AB}$.

$$\mathbf{r}_{AB} = (5\mathbf{i} + 4\mathbf{j})\text{ m}$$

$$\mathbf{T}_{AB} = T_{AB}\frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = T_{AB}(0.781\mathbf{i} + 0.625\mathbf{j})$$

Using the other forces from Active Example 3.5, we have

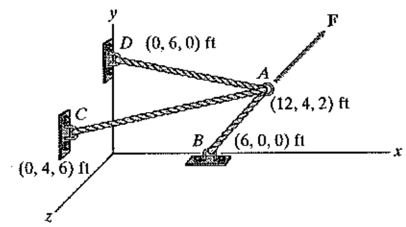
$$\sum F_x : 0.781T_{AB} - 0.408T_{AC} - 0.514T_{AD} = 0$$

$$\sum F_y : 0.625T_{AB} + 0.816T_{AC} + 0.686T_{AD} - 981\text{ N} = 0$$

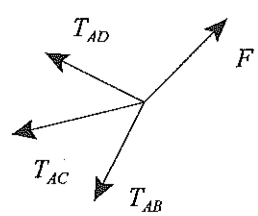
$$\sum F_z : -0.408T_{AC} + 0.514T_{AD} = 0$$

Solving yields $\boxed{T_{AB} = 509\text{ N},\ T_{AC} = 487\text{ N},\ T_{AD} = 386\text{ N}}$

Problem 3.64 The force $\mathbf{F} = 800\mathbf{i} + 200\mathbf{j}$ (lb) acts at point A where the cables AB, AC, and AD are joined. What are the tensions in the three cables?



Solution: We first write the position vectors



$$\mathbf{r}_{AB} = (-6\mathbf{i} - 4\mathbf{j} - 2\mathbf{k})\text{ ft}$$

$$\mathbf{r}_{AC} = (-12\mathbf{i} + 6\mathbf{k})\text{ ft}$$

$$\mathbf{r}_{AD} = (-12\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})\text{ ft}$$

Now we can use these vectors to define the force vectors

$$\mathbf{T}_{AB} = T_{AB}\frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = T_{AB}(-0.802\mathbf{i} - 0.535\mathbf{j} - 0.267\mathbf{k})$$

$$\mathbf{T}_{AC} = T_{AC}\frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = T_{AC}(-0.949\mathbf{i} + 0.316\mathbf{k})$$

$$\mathbf{T}_{AD} = T_{AD}\frac{\mathbf{r}_{AD}}{|\mathbf{r}_{AD}|} = T_{AD}(-0.973\mathbf{i} + 0.162\mathbf{j} - 0.162\mathbf{k})$$

The equilibrium equations are then

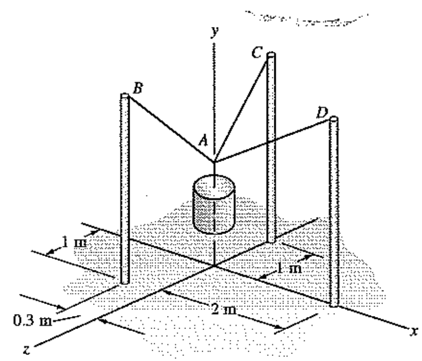
$$\sum F_x : -0.802T_{AB} - 0.949T_{AC} - 0.973T_{AD} + 800\text{ lb} = 0$$

$$\sum F_y : -0.535T_{AB} + 0.162T_{AD} + 200\text{ lb} = 0$$

$$\sum F_z : -0.267T_{AB} + 0.316T_{AC} - 0.162T_{AD} = 0$$

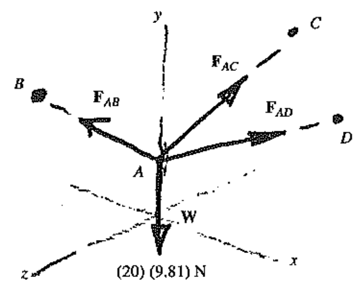
Solving, we find $\boxed{T_{AB} = 405\text{ lb}, T_{AC} = 395\text{ lb}, T_{AD} = 103\text{ lb}}$

Problem 3.69 The 20-kg mass is suspended by cables attached to three vertical 2-m posts. Point A is at (0, 1.2, 0) m. Determine the tensions in cables AB, AC, and AD.



Solution: Points A, B, C, and D are located at

$A(0, 1.2, 0)$, $B(-0.3, 2, 1)$,
$C(0, 2, -1)$, $D(2, 2, 0)$



Write the unit vectors $\mathbf{e}_{AB}, \mathbf{e}_{AC}, \mathbf{e}_{AD}$

$$\mathbf{e}_{AB} = -0.228\mathbf{i} + 0.608\mathbf{j} + 0.760\mathbf{k}$$

$$\mathbf{e}_{AC} = 0\mathbf{i} + 0.625\mathbf{j} - 0.781\mathbf{k}$$

$$\mathbf{e}_{AD} = 0.928\mathbf{i} + 0.371\mathbf{j} + 0\mathbf{k}$$

The forces are

$$\mathbf{F}_{AB} = -0.228F_{AB}\mathbf{i} + 0.608F_{AB}\mathbf{j} + 0.760F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = 0F_{AC}\mathbf{i} + 0.625F_{AC}\mathbf{j} - 0.781F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = 0.928F_{AD}\mathbf{i} + 0.371F_{AD}\mathbf{j} + 0\mathbf{k}$$

$$\mathbf{W} = -(20)(9.81)\mathbf{j}$$

The equations of equilibrium are

$$\begin{cases} \sum F_x = -0.228F_{AB} + 0 + 0.928F_{AD} = 0 \\ \sum F_y = 0.608F_{AB} + 0.625F_{AC} + 0.371F_{AD} - 20(9.81) = 0 \\ \sum F_z = 0.760F_{AB} - 0.781F_{AC} + 0 = 0 \end{cases}$$

We have 3 eqns in 3 unknowns solving, we get

$$\boxed{\begin{array}{l} F_{AB} = 150.0 \text{ N} \\ F_{AC} = 146.1 \text{ N} \\ F_{AD} = 36.9 \text{ N} \end{array}}$$