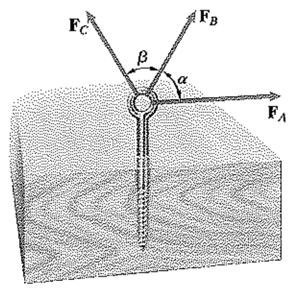
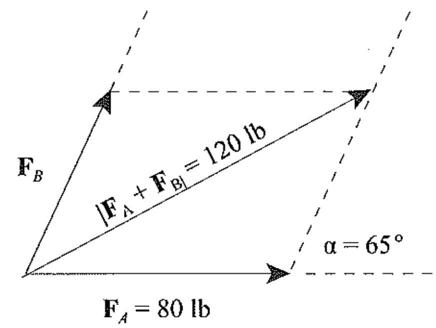
Problem 2.3 The magnitude $|\mathbf{F}_A| = 80$ lb and the angle $\alpha = 65°$. The magnitude $|\mathbf{F}_A + \mathbf{F}_B| = 120$ lb. Graphically determine the magnitude of $\mathbf{F}_B$.

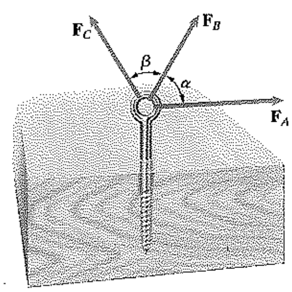


Solution: Accurately draw the vectors and measure the magnitude of $\mathbf{F}_B$.

$$|\mathbf{F}_B| = 62 \text{ lb}$$

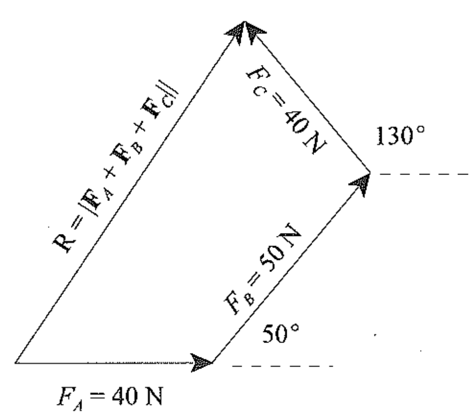


Problem 2.4 The magnitudes $|\mathbf{F}_A| = 40$ N, $|\mathbf{F}_B| = 50$ N, and $|\mathbf{F}_C| = 40$ N. The angle $\alpha = 50°$ and $\beta = 80°$. Graphically determine the magnitude of $\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C$.

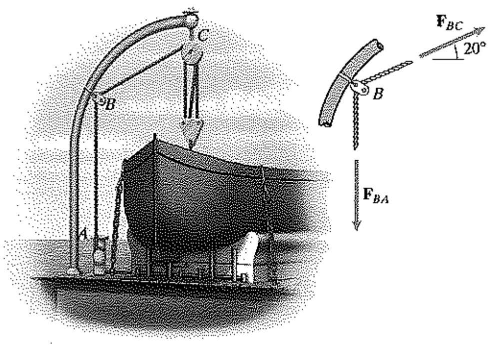


Solution: Accurately draw the vectors and measure the magnitude of $\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C$.

$$R = |\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C| = 83 \text{ N}$$

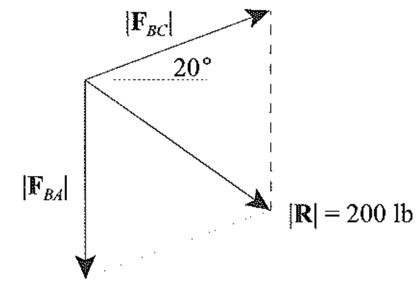


Problem 2.12 The rope *ABC* exerts forces $\mathbf{F}_{BA}$ and $\mathbf{F}_{BC}$ of equal magnitude on the block at *B*. The magnitude of the total force exerted on the block by the two forces is 200 lb. Graphically determine $|\mathbf{F}_{BA}|$.

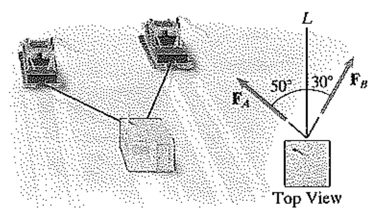


Solution: Draw the vectors accurately and then measure the unknown magnitudes.

$|\mathbf{F}_{BA}| = 174$ lb

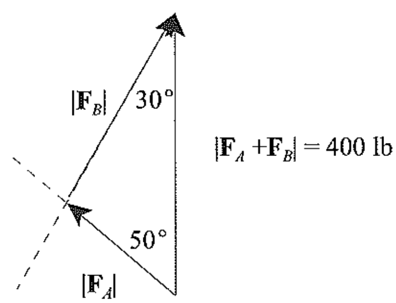


Problem 2.13 Two snowcats tow an emergency shelter to a new location near McMurdo Station, Antarctica. (The top view is shown. The cables are horizontal.) The total force $\mathbf{F}_A + \mathbf{F}_B$ exerted on the shelter is in the direction parallel to the line *L* and its magnitude is 400 lb. Graphically determine the magnitudes of F_A and $\mathbf{F}_B$.

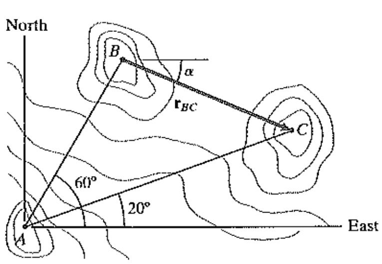


Solution: Draw the vectors accurately and then measure the unknown magnitudes.

$|\mathbf{F}_A| = 203$ lb
$|\mathbf{F}_B| = 311$ lb

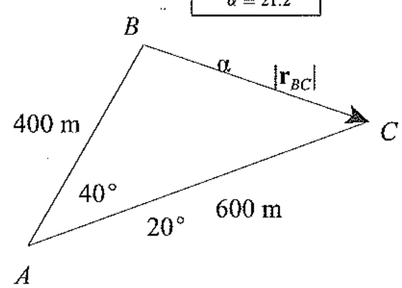


Problem 2.14 A surveyor determines that the horizontal distance from *A* to *B* is 400 m and the horizontal distance from *A* to *C* is 600 m. Graphically determine the magnitude of the vector $\mathbf{r}_{BC}$ and the angle α.

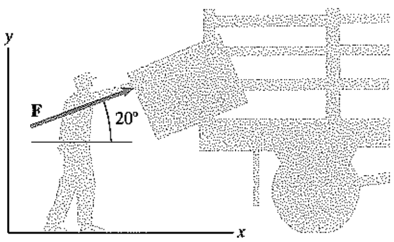


Solution: Draw the vectors accurately and then measure the unknown magnitude and angle.

$|\mathbf{r}_{BC}| = 390$ m
$\alpha = 21.2°$



Problem 2.24 A man exerts a 60-lb force **F** to push a crate onto a truck. (a) Express **F** in terms of components using the coordinate system shown. (b) The weight of the crate is 100 lb. Determine the magnitude of the sum of the forces exerted by the man and the crate's weight.



Solution:

(a) $\mathbf{F} = (60\ \text{lb})(\cos 20^\circ \mathbf{i} + \sin 20^\circ \mathbf{j}) = (56.4\mathbf{i} + 20.5\mathbf{j})\ \text{lb}$

$$\boxed{\mathbf{F} = (56.4\mathbf{i} + 20.5\mathbf{j})\text{lb}}$$

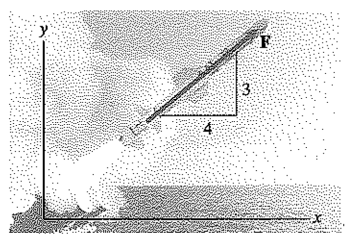
(b) $\mathbf{W} = -(100\ \text{lb})\mathbf{j}$

$$\mathbf{F} + \mathbf{W} = (56.4\mathbf{i} + [20.5 - 100]\mathbf{j})\ \text{lb} = (56.4\mathbf{i} - 79.5\mathbf{j})\ \text{lb}$$

$$|\mathbf{F} + \mathbf{W}| = \sqrt{(56.4\ \text{lb})^2 + (-79.5\ \text{lb})^2} = 97.4\ \text{lb}$$

$$\boxed{|\mathbf{F} + \mathbf{W}| = 97.4\ \text{lb}}$$

Problem 2.25 The missile's engine exerts a 260-kN force **F**. (a) Express **F** in terms of components using the coordinate system shown. (b) The mass of the missile is 8800 kg. Determine the magnitude of the sum of the forces exerted by the engine and the missile's weight.



Solution:

(a) We can use similar triangles to determine the components of **F**.

$$\mathbf{F} = (260\ \text{kN})\left(\frac{4}{\sqrt{4^2+3^2}}\mathbf{i} + \frac{3}{\sqrt{4^2+3^2}}\mathbf{j}\right) = (208\mathbf{i} + 156\mathbf{j})\ \text{kN}$$

$$\boxed{\mathbf{F} = (208\mathbf{i} + 156\mathbf{j})\ \text{kN}}$$

(b) The missile's weight **W** can be expressed in component and then added to the force **F**.

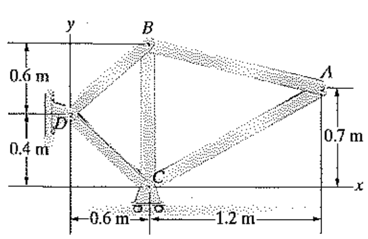
$$\mathbf{W} = -(8800\ \text{kg})(9.81\ \text{m/s}^2)\mathbf{j} = -(86.3\ \text{kN})\mathbf{j}$$

$$\mathbf{F} + \mathbf{W} = (208\mathbf{i} + [156 - 86.3]\mathbf{j})\ \text{kN} = (208\mathbf{i} - 69.7\mathbf{j})\ \text{kN}$$

$$|\mathbf{F} + \mathbf{W}| = \sqrt{(208\ \text{kN})^2 + (-69.7\ \text{kN})^2} = 219\ \text{kN}$$

$$\boxed{|\mathbf{F} + \mathbf{W}| = 219\ \text{kN}}$$

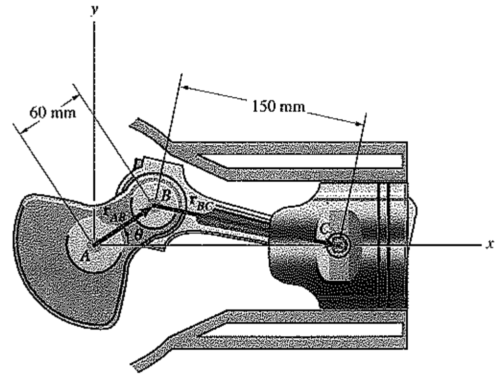
Problem 2.26 For the truss shown, express the position vector $\mathbf{r}_{AD}$ from point A to point D in terms of components. Use your result to determine the distance from point A to point D.



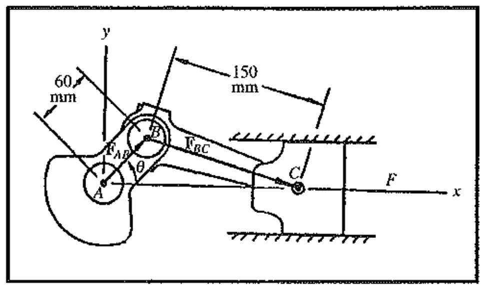
Solution: Coordinates $A(1.8, 0.7)$ m, $D(0, 0.4)$ m

$$\boxed{\begin{array}{l} \mathbf{r}_{AD} = (0 - 1.8\ \text{m})\mathbf{i} + (0.4\ \text{m} - 0.7\ \text{m})\mathbf{j} = (-1.8\mathbf{i} - 0.3\mathbf{j})\ \text{m} \\ r_{AD} = \sqrt{(-1.8\ \text{m})^2 + (-0.3\ \text{m})^2} = 1.825\ \text{m} \end{array}}$$

Problem 2.32 Determine the position vector $\mathbf{r}_{AB}$ in terms of its components if (a) $\theta = 30°$, (b) $\theta = 225°$.



Solution:



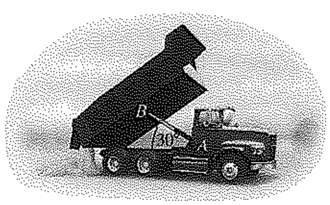
(a) $\mathbf{r}_{AB} = (60)\cos(30°)\mathbf{i} + (60)\sin(30°)\mathbf{j}$, or

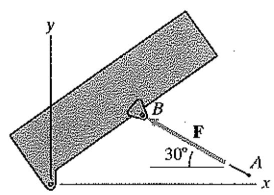
$\mathbf{r}_{AB} = 51.96\mathbf{i} + 30\mathbf{j}$ mm. And

(b) $\mathbf{r}_{AB} = (60)\cos(225°)\mathbf{i} + (60)\sin(225°)\mathbf{j}$ or

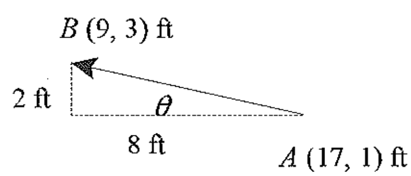
$\mathbf{r}_{AB} = -42.4\mathbf{i} - 42.4\mathbf{j}$ mm.

Problem 2.33 In Example 2.4, the coordinates of the fixed point A are (17, 1) ft. The driver lowers the bed of the truck into a new position in which the coordinates of point B are (9, 3) ft. The magnitude of the force F exerted on the bed by the hydraulic cylinder when the bed is in the new position is 4800 lb. Draw a sketch of the new situation. Express F in terms of components.





Solution:



$$\theta = \tan^{-1}\left(\frac{2\text{ ft}}{8\text{ ft}}\right) = 14.04°$$

$\mathbf{F} = 4800\text{ lb}(-\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$.

$\boxed{\mathbf{F} = (-4660\mathbf{i} + 1160\mathbf{j})\text{ lb}}$

Problem 2.40 The hydraulic actuator BC in Problem 2.39 exerts a 1.2-kN force $\mathbf{F}$ on the joint at C that is parallel to the actuator and points from B toward C. Determine the components of $\mathbf{F}$.

Solution: From the solution to Problem 2.39,

$$\mathbf{e}_{BC} = -0.781\mathbf{i} + 0.625\mathbf{j}$$

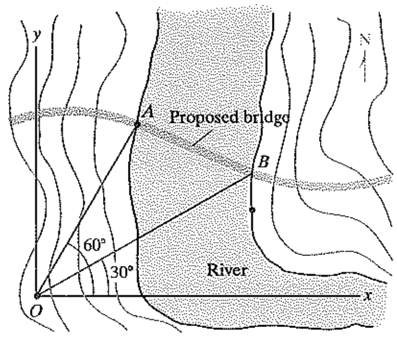
The vector $\mathbf{F}$ is given by $\mathbf{F} = |\mathbf{F}|\mathbf{e}_{BC}$

$$\mathbf{F} = (1.2)(-0.781\mathbf{i} + 0.625\mathbf{j})\ (\mathrm{k \cdot N})$$

$$\mathbf{F} = -937\mathbf{i} + 750\mathbf{j}\ (\mathrm{N})$$

Problem 2.41 A surveyor finds that the length of the line OA is 1500 m and the length of line OB is 2000 m.

(a) Determine the components of the position vector from point A to point B.
(b) Determine the components of a unit vector that points from point A toward point B.



Solution: We need to find the coordinates of points A and B

$$\mathbf{r}_{OA} = 1500\cos 60^\circ\mathbf{i} + 1500\sin 60^\circ\mathbf{j}$$

$$\mathbf{r}_{OA} = 750\mathbf{i} + 1299\mathbf{j}\ (\mathrm{m})$$

Point A is at (750, 1299) (m)

$$\mathbf{r}_{OB} = 2000\cos 30^\circ\mathbf{i} + 2000\sin 30^\circ\mathbf{j}\ (\mathrm{m})$$

$$\mathbf{r}_{OB} = 1732\mathbf{i} + 1000\mathbf{j}\ (\mathrm{m})$$

Point B is at (1732, 1000) (m)

(a) The vector from A to B is

$$\mathbf{r}_{AB} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j}$$

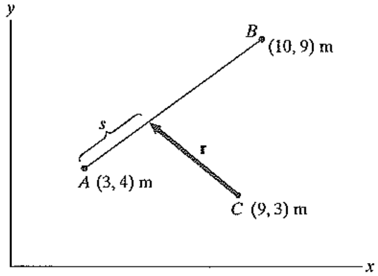
$$\mathbf{r}_{AB} = 982\mathbf{i} - 299\mathbf{j}\ (\mathrm{m})$$

(b) The unit vector $\mathbf{e}_{AB}$ is

$$\mathbf{e}_{AB} = \frac{r_{AB}}{|\mathbf{r}_{AB}|} = \frac{982\mathbf{i} - 299\mathbf{j}}{1026.6}$$

$$\mathbf{e}_{AB} = 0.957\mathbf{i} - 0.291\mathbf{j}$$

Problem 2.60 Let $\mathbf{r}$ be the position vector from point C to the point that is a distance s meters along the straight line between A and B. Express $\mathbf{r}$ in terms of components. (Your answer will be in terms of s).



Solution: First define the unit vector that points from A to B.

$$\mathbf{r}_{B/A} = ([10-3]\mathbf{i} + [9-4]\mathbf{j})\ m = (7\mathbf{i} + 5\mathbf{j})\ \text{m}$$

$$|\mathbf{r}_{B/A}| = \sqrt{(7\ \text{m})^2 + (5\ \text{m})^2} = \sqrt{74}\ \text{m}$$

$$\mathbf{e}_{B/A} = \frac{1}{\sqrt{74}}(7\mathbf{i} + 5\mathbf{j})$$

Let P be the point that is a distance s along the line from A to B. The coordinates of point P are

$$x_p = 3\ \text{m} + s\left(\frac{7}{\sqrt{74}}\right) = (3 + 0.814s)\ \text{m}$$

$$y_p = 4\ \text{m} + s\left(\frac{5}{\sqrt{74}}\right) = (4 + 0.581s)\ \text{m}.$$

The vector $\mathbf{r}$ that points from C to P is then

$$\mathbf{r} = ([3 + 0.814s - 9]\mathbf{i} + [4 + 0.581s - 3]\mathbf{j})\ \text{m}$$

$$\boxed{\mathbf{r} = ([0.814s - 6]\mathbf{i} + [0.581s + 1]\mathbf{j})\ \text{m}}$$

Problem 2.61 A vector $\mathbf{U} = 3\mathbf{i} - 4\mathbf{j} - 12\mathbf{k}$. What is its magnitude?

Strategy: The magnitude of a vector is given in terms of its components by Eq. (2.14).

Solution: Use definition given in Eq. (14). The vector magnitude is

$$\boxed{|\mathbf{U}| = \sqrt{3^2 + (-4)^2 + (-12)^2} = 13}$$

Problem 2.62 The vector $\mathbf{e} = \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + e_z\mathbf{k}$ is a unit vector. Determine the component e_z. (Notice that there are two answers.)

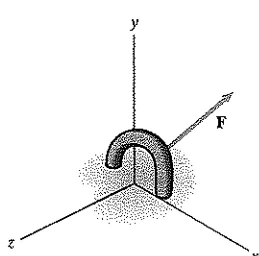
Solution:

$$\mathbf{e} = \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + e_z\mathbf{k} \Rightarrow \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + e_z{}^2 = 1 \Rightarrow e^2 = \frac{4}{9}$$

Thus

$$\boxed{e_z = \frac{2}{3} \quad \text{or} \quad e_z = -\frac{2}{3}}$$

Problem 2.63 An engineer determines that an attachment point will be subjected to a force $\mathbf{F} = 20\mathbf{i} + F_y\mathbf{j} - 45\mathbf{k}$ (kN). If the attachment point will safely support a force of 80-kN magnitude in any direction, what is the acceptable range of values for F_y?



Solution:

$$80^2 \geq F_x^2 + F_y^2 + F_z^2$$

$$80^2 \geq 20^2 + F_y^2 + (45)^2$$

To find limits, use equality.

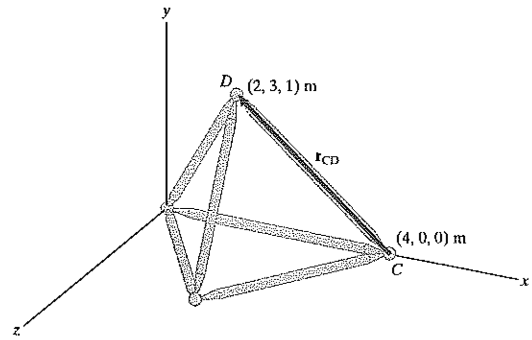
$$F_{y\text{LIMIT}}^2 = 80^2 - 20^2 - (45)^2$$

$$F_{y\text{LIMIT}}^2 = 3975$$

$$F_{y\text{LIMIT}} = +63.0, -63.0\ \text{(kN)}$$

$$|F_{y\text{LIMIT}}| \leq 63.0\ \text{kN} - 63.0\ \text{kN} \leq F_y \leq 63.0\ \text{kN}$$

Problem 2.67 In Active Example 2.6, suppose that you want to redesign the truss, changing the position of point D so that the magnitude of the vector $\mathbf{r}_{CD}$ from point C to point D is 3 m. To accomplish this, let the coordinates of point D be (2, y_D, 1) m, and determine the value of y_D so that $|\mathbf{r}_{CD}| = 3$ m. Draw a sketch of the truss with point D in its new position. What are the new directions cosines of $\mathbf{r}_{CD}$?



(a)

Solution: The vector $\mathbf{r}_{CD}$ and the magnitude $|\mathbf{r}_{CD}|$ are

$$\mathbf{r}_{CD} = ([2\text{ m} - 4\text{ m}]\mathbf{i} + [y_D - 0]\mathbf{j} + [1\text{ m} - 0]\mathbf{k}) = (-2\text{ m})\mathbf{i} + (y_D)\mathbf{j} + (1\text{ m})\mathbf{k}$$

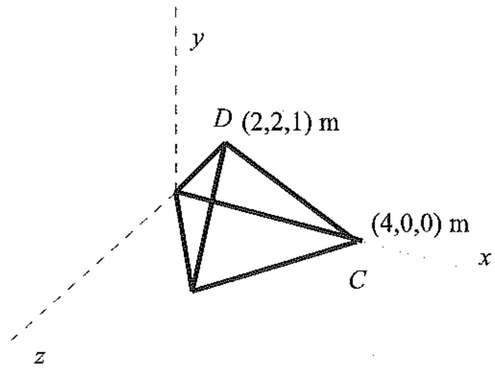
$$|\mathbf{r}_{CD}| = \sqrt{(-2\text{ m})^2 + (y_{CD})^2 + (1\text{ m})^2} = 3\text{ m}$$

Solving we find $y_{CD} = \sqrt{(3\text{ m})^2 - (-2\text{ m})^2 - (1\text{ m})^2} = 2$ m

$\boxed{y_{CD} = 2\text{ m}}$

The new direction cosines of $\mathbf{r}_{CD}$.

$$\boxed{\begin{aligned}\cos\theta_x &= -2/3 = -0.667\\ \cos\theta_y &= 2/3 = 0.667\\ \cos\theta_z &= 1/3 = 0.333\end{aligned}}$$



Problem 2.68 A force vector is given in terms of its components by $\mathbf{F} = 10\mathbf{i} - 20\mathbf{j} - 20\mathbf{k}$ (N).

(a) What are the direction cosines of $\mathbf{F}$?
(b) Determine the components of a unit vector $\mathbf{e}$ that has the same direction as $\mathbf{F}$.

Solution:

$$\mathbf{F} = (10\mathbf{i} - 20\mathbf{j} - 20\mathbf{k})\text{ N}$$

$$F = \sqrt{(10\text{ N})^2 + (-20\text{ N})^2 + (-20\text{ N})^2} = 30\text{ N}$$

(a) $$\boxed{\cos\theta_x = \frac{10\text{ N}}{30\text{ N}} = 0.333,\quad \cos\theta_y = \frac{-20\text{ N}}{30\text{ N}} = -0.667,\quad \cos\theta_z = \frac{-20\text{ N}}{30\text{ N}} = -0.667}$$

(b) $\boxed{\mathbf{e} = (0.333\mathbf{i} - 0.667\mathbf{j} - 0.667\mathbf{k})}$