NAME DATE

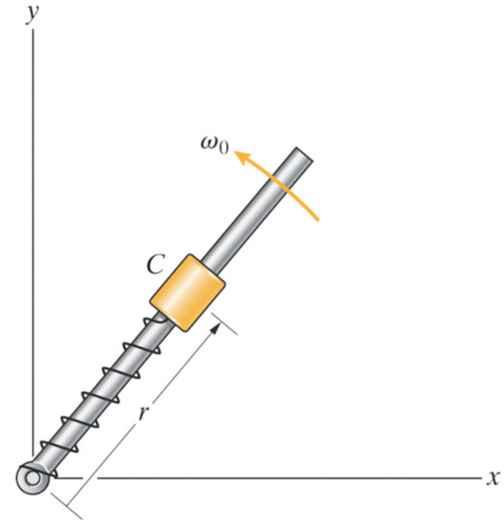
WEEK: ______ **PROBLEM:** ______

GIVEN:

The bar rotates in the $x-y$ plane with constant angular velocity $\omega_0 = 12$ rad/s. The radial component of acceleration of the collar C (in m/s^2) is given as a function of the radial position in meters by $a_r = -8r$. When $r = 1$ m, the radial component of velocity of C is $v_r = 2$ m/s. Determine the velocity of C in terms of polar coordinates when $r = 1.5$ m.

Strategy: Use the chain rule to write the first term in the radial component of the acceleration as

$$\frac{d^2r}{dt^2} = \frac{dv_r}{dt} = \frac{dv_r}{dr}\frac{dr}{dt} = \frac{dv_r}{dr}v_r$$



REQUIRED:

SOLUTION:

Solution: We have

$$a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -(8 \text{ rad/s}^2)r,$$

$$\frac{d^2r}{dt^2} = \left(\left[\frac{d\theta}{dt}\right]^2 - (8 \text{ rad/s}^2)\right)r = ([12 \text{ rad/s}]^2 - [8 \text{ rad/s}^2])r = ([136 \text{ rad/s}^2)r$$

Using the supplied strategy we can solve for the radial velocity

$$\frac{d^2r}{dt^2} = v_r\frac{dv_r}{dr} = (136 \text{ rad/s}^2)r$$

$$\int_{2 \text{ m/s}}^{v_r} v_r\,dv_r = (136 \text{ rad/s}^2)\int_{1 \text{ m}}^{1.5 \text{ m}} r\,dr$$

$$\frac{v_r^2}{2} - \frac{(2 \text{ m/s})^2}{2} = (136 \text{ rad/s}^2)\left(\frac{[1.5 \text{ m}]^2}{2} - \frac{[1 \text{ m}]^2}{2}\right)$$

Solving we find $v_r = 13.2$ m/s.

We also have $v_\theta = r\dfrac{d\theta}{dt} = (1.5 \text{ m})(12 \text{ rad/s}) = 18$ m/s.

Thus $\boxed{\mathbf{v} = (13.2\mathbf{e}_r + 18\mathbf{e}_\theta) \text{ m/s.}}$