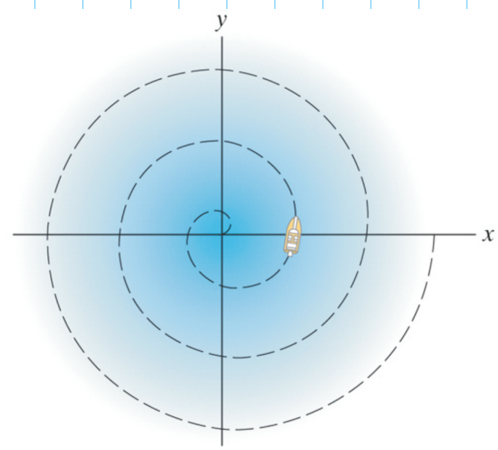
NAME DATE

WEEK: ______ PROBLEM: ______

GIVEN:

A boat searching for underwater archaeological sites in the Aegean Sea moves at 4 knots and follows the path $r = 10\theta$ m, where θ is in radians. (A knot is one nautical mile, or 1852 meters, per hour.) When $\theta = 2\pi$ rad, determine the boat's velocity (a) in terms of polar coordinates and (b) in terms of cartesian coordinates.



REQUIRED:

SOLUTION:

Solution: The velocity along the path is

$$v = 4\left(\frac{1852 \text{ m}}{1 \text{ knot}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 2.06 \text{ m/s}.$$

(a) The path is $r = 10\theta$. The velocity

$$v_r = \frac{dr}{dt} = \frac{d}{dt}(10\theta) = 10\frac{d\theta}{dt} \text{ m/s}.$$

The velocity along the path is related to the components by

$$v^2 = v_r^2 + v_\theta^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 = 2.06^2.$$

At $\theta = 2\pi$, $r = 10(2\pi) = 62.8$ m. Substitute:

$$2.06^2 = \left(10\frac{d\theta}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 = (100 + 62.8^2)\left(\frac{d\theta}{dt}\right)^2,$$

from which $\frac{d\theta}{dt} = 0.0323$ rad/s,

$$\boxed{v_r = 10\frac{d\theta}{dt} = 0.323 \text{ m/s}}, \quad \boxed{v_\theta = r\frac{d\theta}{dt} = 2.032 \text{ m/s}}$$

(b) From geometry, the cartesian components are $v_x = v_r \cos\theta + v_\theta \sin\theta$, and $v_y = v_r \sin\theta + v_\theta \cos\theta$. At $\theta = 2\pi$,

$\boxed{v_x = v_r}$, and $\boxed{v_y = v_\theta}$