Name

Date

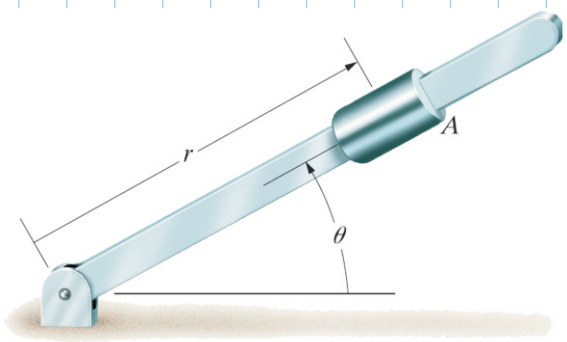
Week: _____ Problem: _____

Given:

The polar coordinates of the collar A are given as functions of time in seconds by

$$r = 1 + 0.2t^2 \text{ ft and } \theta = 2t \text{ rad.}$$

What are the magnitudes of the velocity and acceleration of the collar at $t = 2$ s?



Required:

Solution:

Solution: We have

$r = (1 \text{ ft}) + (0.2 \text{ ft/s}^2)t^2, \quad \theta = (2 \text{ rad/s})t,$

$\frac{dr}{dt} = (0.4 \text{ ft/s}^2)t, \quad \frac{d\theta}{dt} = 2 \text{ rad/s},$

$\frac{d^2r}{dt^2} = 0.4 \text{ ft/s}^2, \quad \frac{d^2\theta}{dt^2} = 0$

At time $t = 2$ s, we have

$r = 1.8 \text{ ft}, \quad \frac{dr}{dt} = 0.8 \text{ ft/s}, \quad \frac{d^2r}{dt^2} = 0.4 \text{ ft/s}^2,$

$\theta = 8 \text{ rad}, \quad \frac{d\theta}{dt} = 2 \text{ rad/s}, \quad \frac{d^2\theta}{dt^2} = 0$

The components of the velocity and acceleration are

$v_r = \frac{dr}{dt} = 0.8 \text{ ft/s}, \; v_\theta = r\frac{d\theta}{dt} = (1.8 \text{ ft})(2 \text{ rad/s}) = 3.6 \text{ ft/s},$

$a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = (0.4 \text{ ft/s}^2) - (1.8 \text{ ft})(2 \text{ rad/s})^2 = -6.8 \text{ ft/s}^2,$

$a_\theta = r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt} = 0 + 2(0.8 \text{ ft/s})(2 \text{ rad/s}) = 3.2 \text{ ft/s}^2.$

The magnitudes are

$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(0.8 \text{ ft/s})^2 + (3.6 \text{ ft/s})^2} = 3.69 \text{ ft/s},$

$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-6.8 \text{ ft/s}^2)^2 + (3.2 \text{ ft/s}^2)^2} = 7.52 \text{ ft/s}^2.$

$\boxed{v = 3.69 \text{ ft/s}, a = 7.52 \text{ ft/s}^2.}$