Chapter 3

Particle Dynamics: Work and Energy

Kinetic Energy

Energy is the ability to do **work**. Imagine that you throw a particle of mass m right upward with an initial speed v. You've transferred some energy to the particle. How much is the energy? To answer this question, let's calculate how much **work** this energy can do. The particle moves upward until a height h,

$$h = \frac{v^2}{2g}$$

the work done is

$$W = mgh = mg\frac{v^2}{2g} = \frac{mv^2}{2}$$

We conclude that *a moving particle with a speed of v possesses an energy of* $mv^2/2$. This energy is defined as the **kinetic energy** T of the particle,

$$T = \frac{mv^2}{2} \qquad (1)$$

Principle of Work and Energy

Consider a particle of mass m acted upon by a force $\vec{F}$ and moving along a path. Applying Eq. 2(2) (page 34), in the tangential direction, we write

$$F_t = ma_t = m\frac{dv}{dt} = m\frac{dv}{ds}\frac{ds}{dt} = mv\frac{dv}{ds}$$

or

$$F_t ds = mvdv \qquad (2)$$

where v is the speed of the particle and s is the accumulative length along the path. Integrating both sides of Eq. (2) for any duration from t_1 to t_2, where the accumulative lengths are s_1 and s_2 and the speeds of the particle are v_1 and v_2, we write

$$\int_{s_1}^{s_2} F_t\, ds = \int_{v_1}^{v_2} mv\, dv = \frac{mv_2^2}{2} - \frac{mv_1^2}{2} = T_2 - T_1 \qquad (3)$$

where T_1 and T_2 are the kinetic energy possessed by the particle at t_1 and t_2 respectively. The left hand side $\int_{s_1}^{s_2} F_t\, ds$ is the **work done** $U_{1\to 2}$ by the force during t_1 and t_2. We may rewrite Eq. (3) in the form

$$T_1 + U_{1\to 2} = T_2 \qquad (4)$$

Eq. (4) is called the **principle of work and energy**.

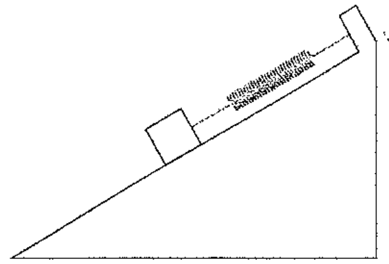
Work done by **conservative forces** (e.g., gravitational force or spring force, by which the work done is independent of path) can be expressed in terms of **potential energy** V. Incorporating potential energy into Eq. (4), we may write the **principle of work and energy** in a more useful form:

$$T_1 + V_1 + U_{1\to 2} = T_2 + V_2 \qquad (5)$$

This chapter will show how Eq. (5) is satisfied in a particle system.

Section 3.1

Principle of Work and Energy: Oscillating Block



3.1-1 Introduction

[1] Consider a **Block** [2] connected to a **Ground** [3] with a Spring [4], sliding along a 30° slope. The **Block** is initially positioned such that the **Spring** has an initial elongation of 30 cm [5]. The **Dynamic Friction Coefficient** between the **Block** and the **Ground** is 0.3 [6].

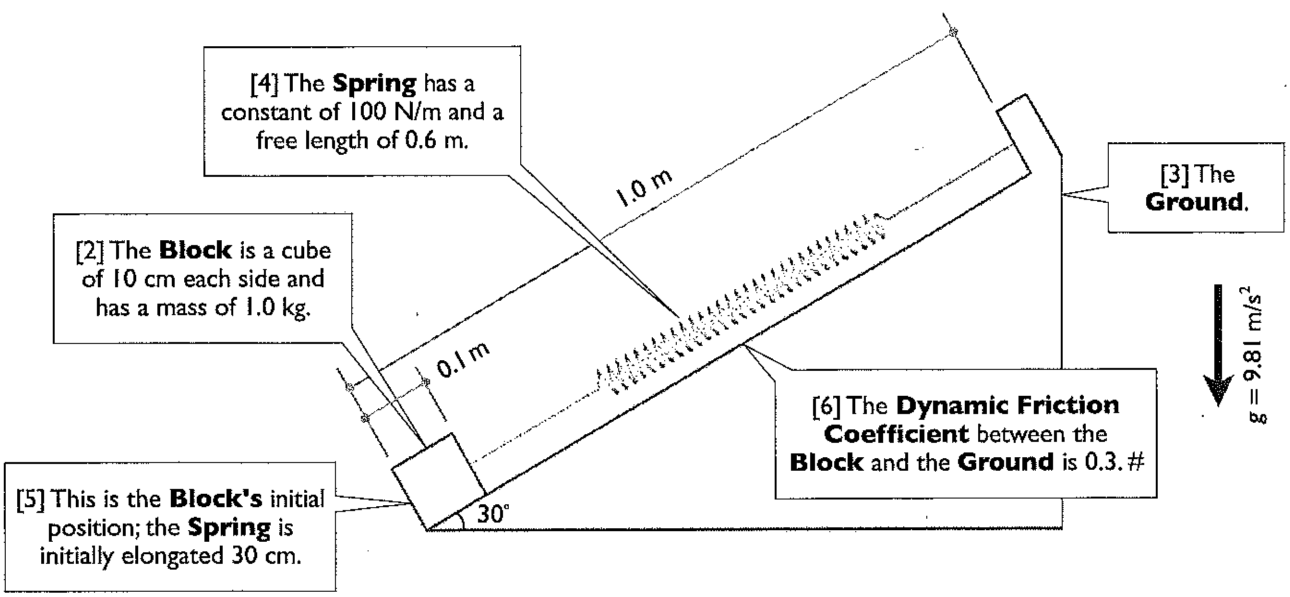
Using this example, we'll illustrate the **principle of work and energy**, which states

$$T_0 + V_0 + U_{0\to t} = T_t + V_t \tag{1}$$

where T_0 and V_0 are respectively the initial kinetic energy and potential energy of the system, and T_t and V_t are respectively the kinetic energy and potential energy of the system at time t, and $U_{0\to t}$ is the work done by non-conservative force (work done by conservative forces always can be expressed as potential energies). In this case $T_0 = 0$ and $V_0 = (100\ \text{N/m})(0.3\ \text{m})^2/2 = 4.5\ \text{J}$ (the initial position is taken as the baseline of the gravitational potential energy); therefore, Eq. (1) can be rewritten as

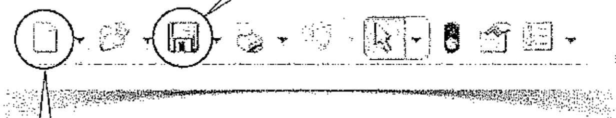
$$T_t + V_t - U_{0\to t} = 4.5\ \text{J} \tag{2}$$

In other words, the **principle of work and energy** can be restated as follows: *at any time, the sum of kinetic energy and potential energy, minus the work done by non-conservative forces (in this case, the friction forces), remains a constant.* The statement can be viewed as a form of **conservation of energy**.

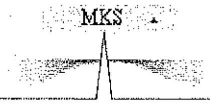


3.1-2 Start Up and Create a Part: **Block**

[2] Save the **Part** with the name **Block**.

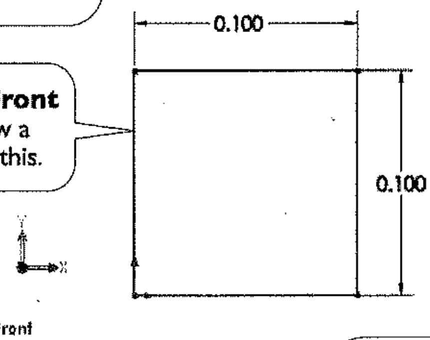


[1] Launch **SOLIDWORKS** and click **New** to create a new **Part**.

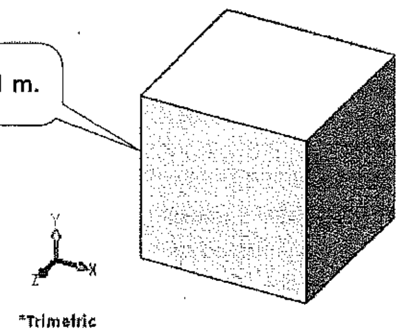


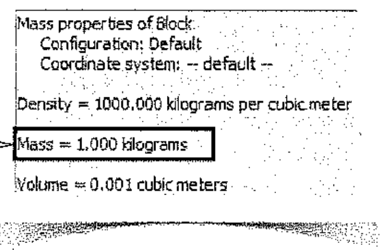
[3] Set up **MKS** unit system with three decimal places for **Length**.

[4] On the **Front** plane, draw a square like this.



[5] Extrude 0.1 m.





Mass properties of Block
Configuration: Default
Coordinate system: -- default --

Density = 1000.000 kilograms per cubic meter

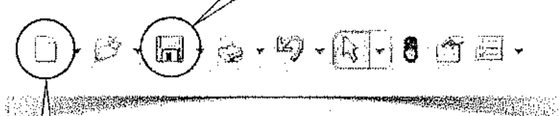
Mass = 1.000 kilograms

Volume = 0.001 cubic meters

[6] In the **Evaluate Toolbar**, click **Mass Properties** and make sure the mass of the **Block** is 1.000 kg. #

3.1-3 Create a Part: **Ground**

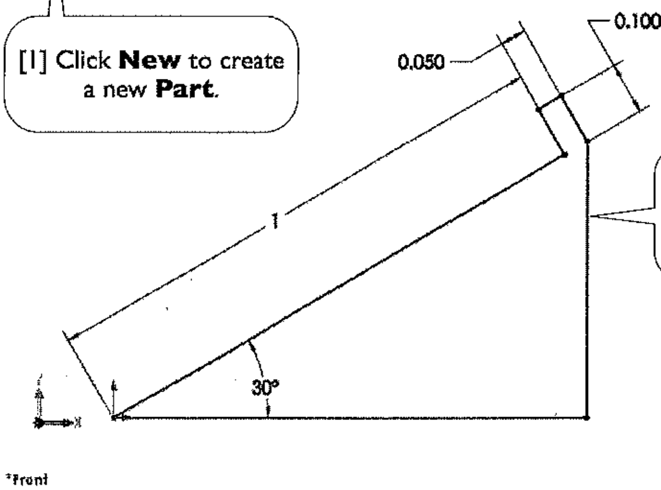
[2] Save the **Part** with the name **Ground**.



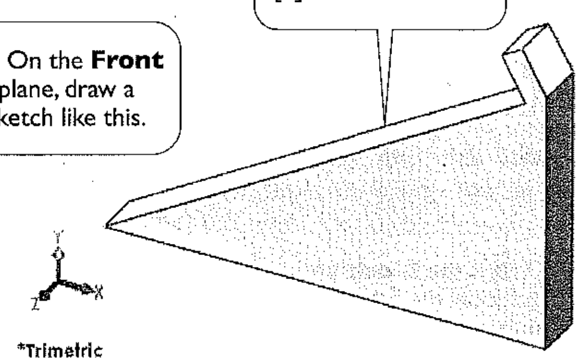
[1] Click **New** to create a new **Part**.

[3] Set up **MKS** unit system with three decimal places for **Length**.

[4] On the **Front** plane, draw a sketch like this.



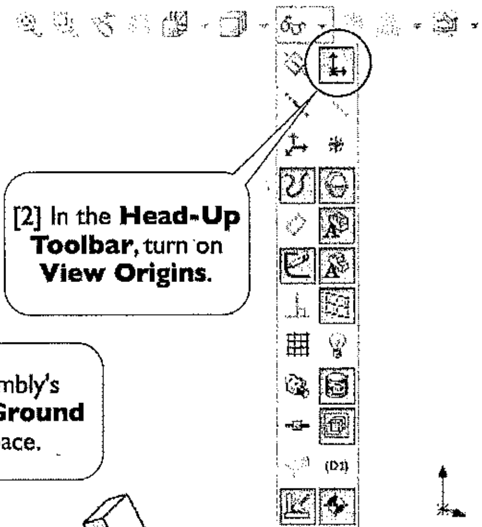
[5] Extrude 0.1 m. #



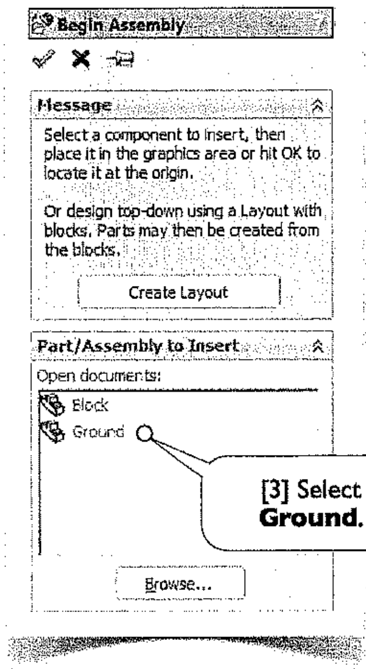
3.1-4 Create an Assembly: **Oscillating-Block**

[1] Click **New** and create an **Assembly**.

[5] Save the **Assembly** with the name **Oscillating-Block**.

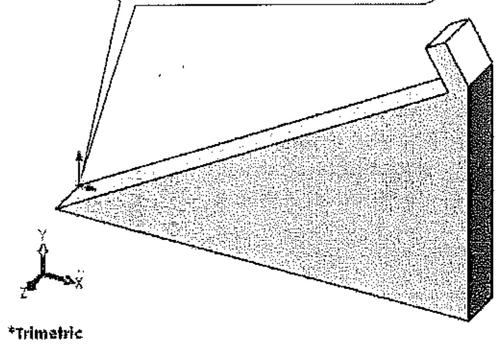


[2] In the **Head-Up Toolbar**, turn on **View Origins**.

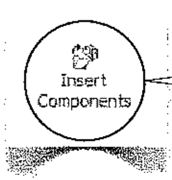


[3] Select **Ground.**

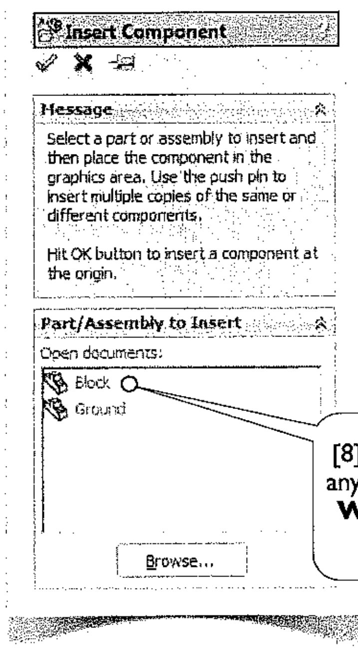
[4] Click the assembly's **Origin**. Now the **Ground** is fixed in the space.



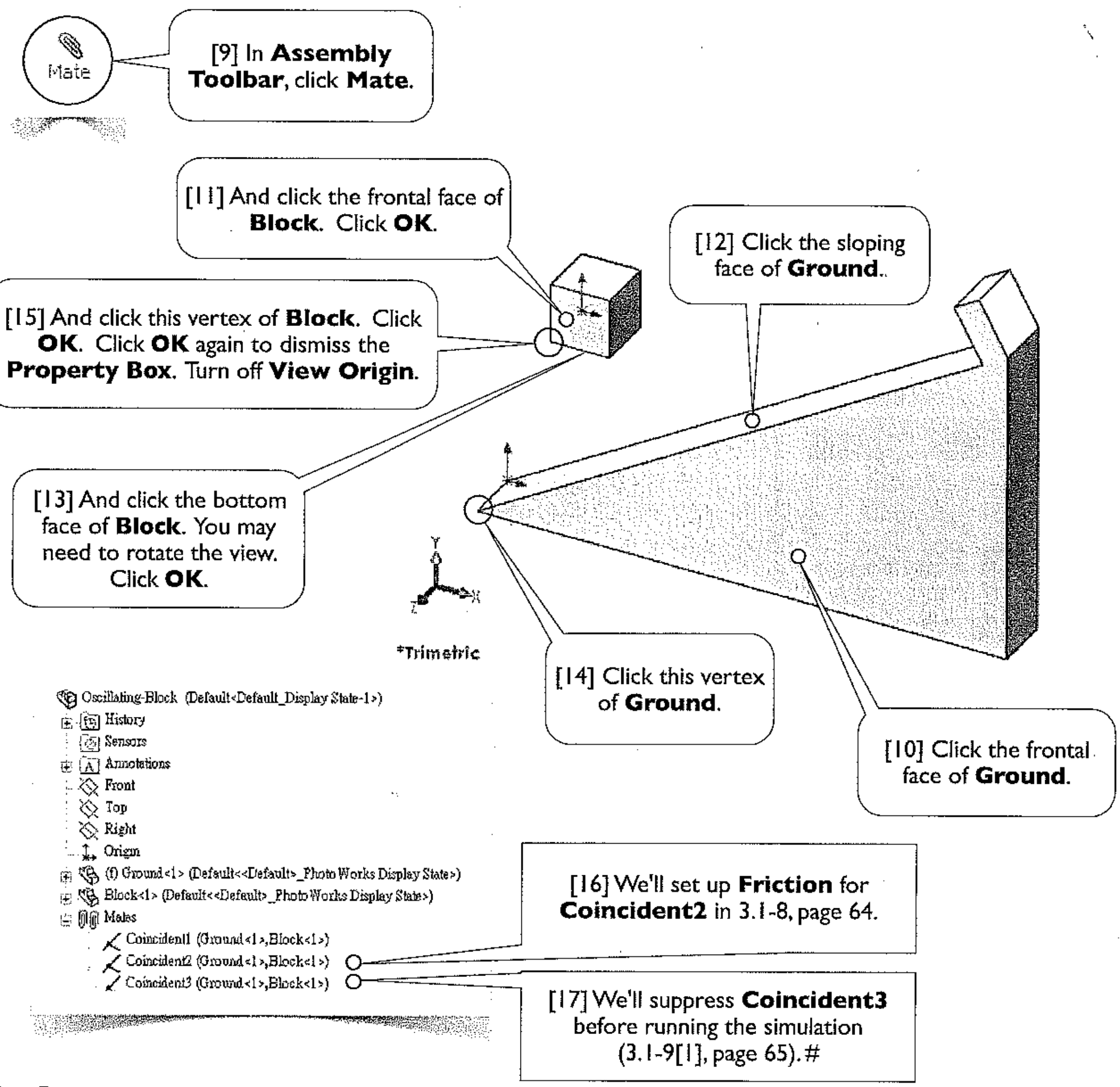
[6] Set up **MKS** unit system with three decimal places for **Length**.



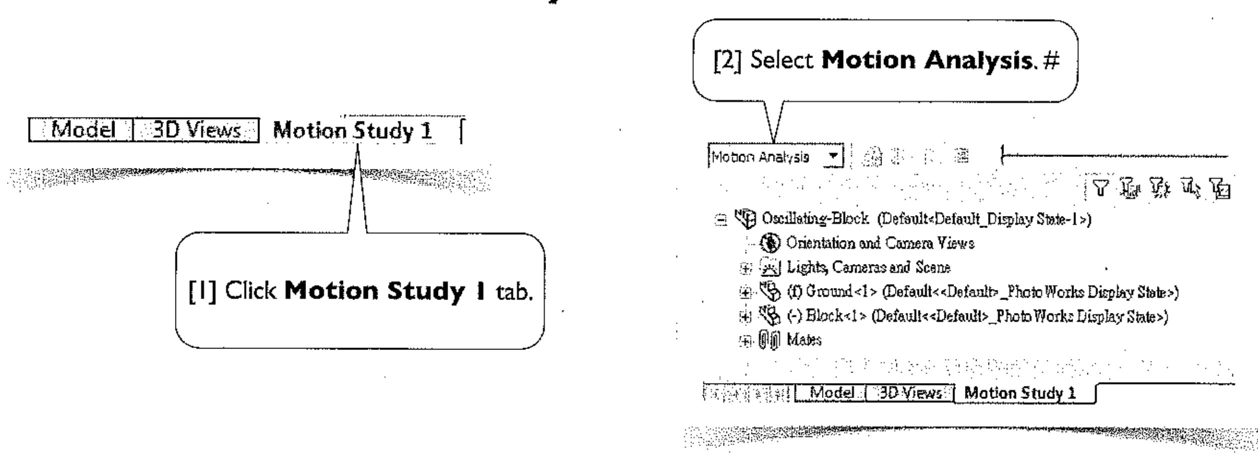
[7] In the **Assembly Toolbar**, click **Insert Components**.



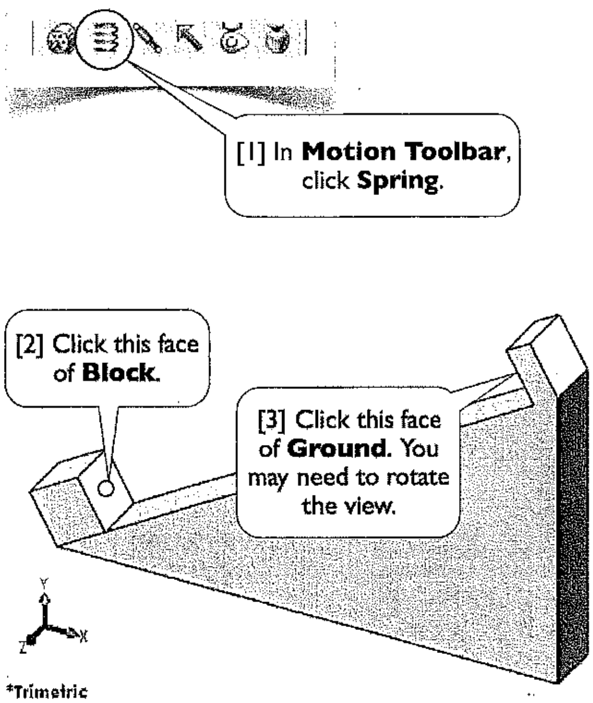
[8] Select **Block** and click anywhere in the **Graphics Window** to temporarily park the part.

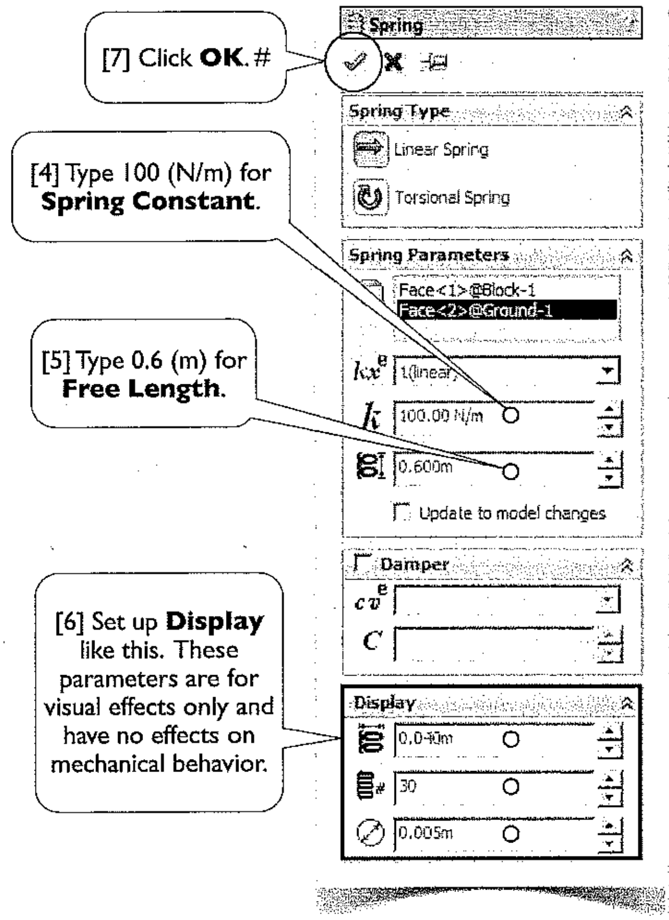


3.1-5 Create a **Motion Study**

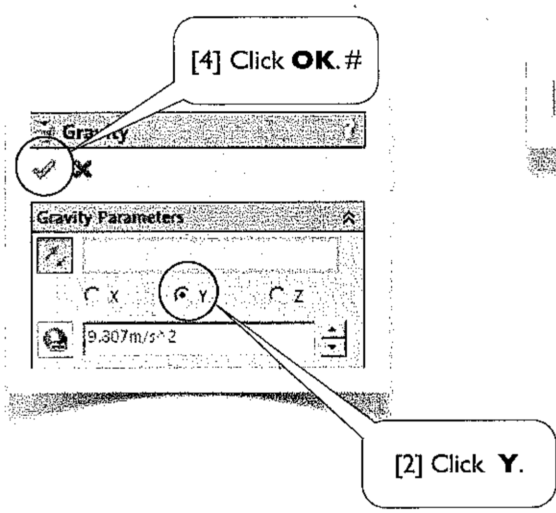


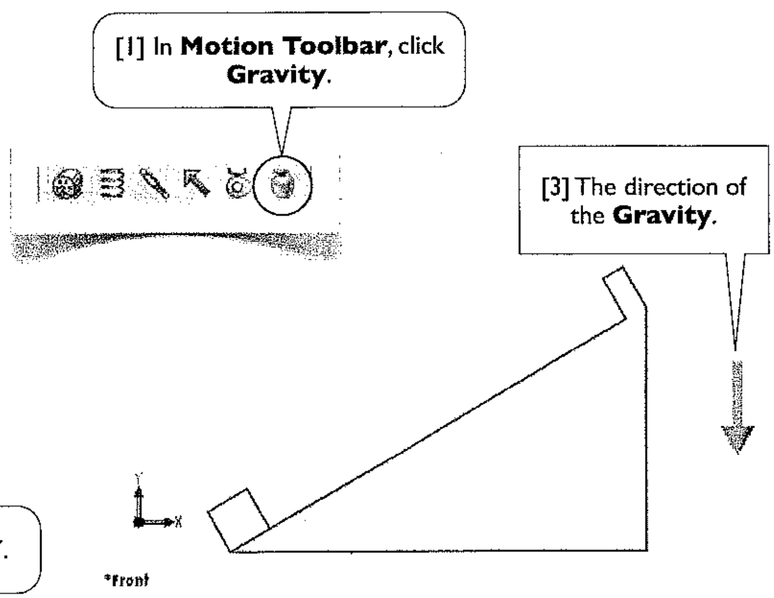
3.1-6 Set Up **Spring**



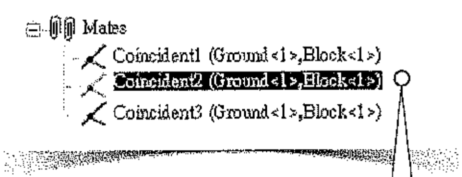


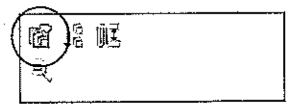
3.1-7 Set Up **Gravity**



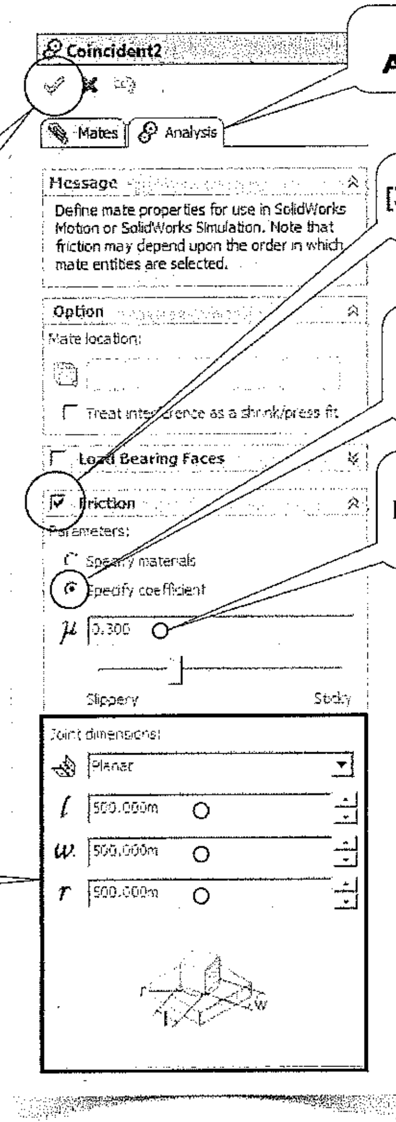


3.1-8 Set Up **Friction**





[1] In the **Assembly Tree**, right-click **Coincident2** and select **Edit Feature**.



[2] Click **Analysis** tab.

[3] Click **Friction**.

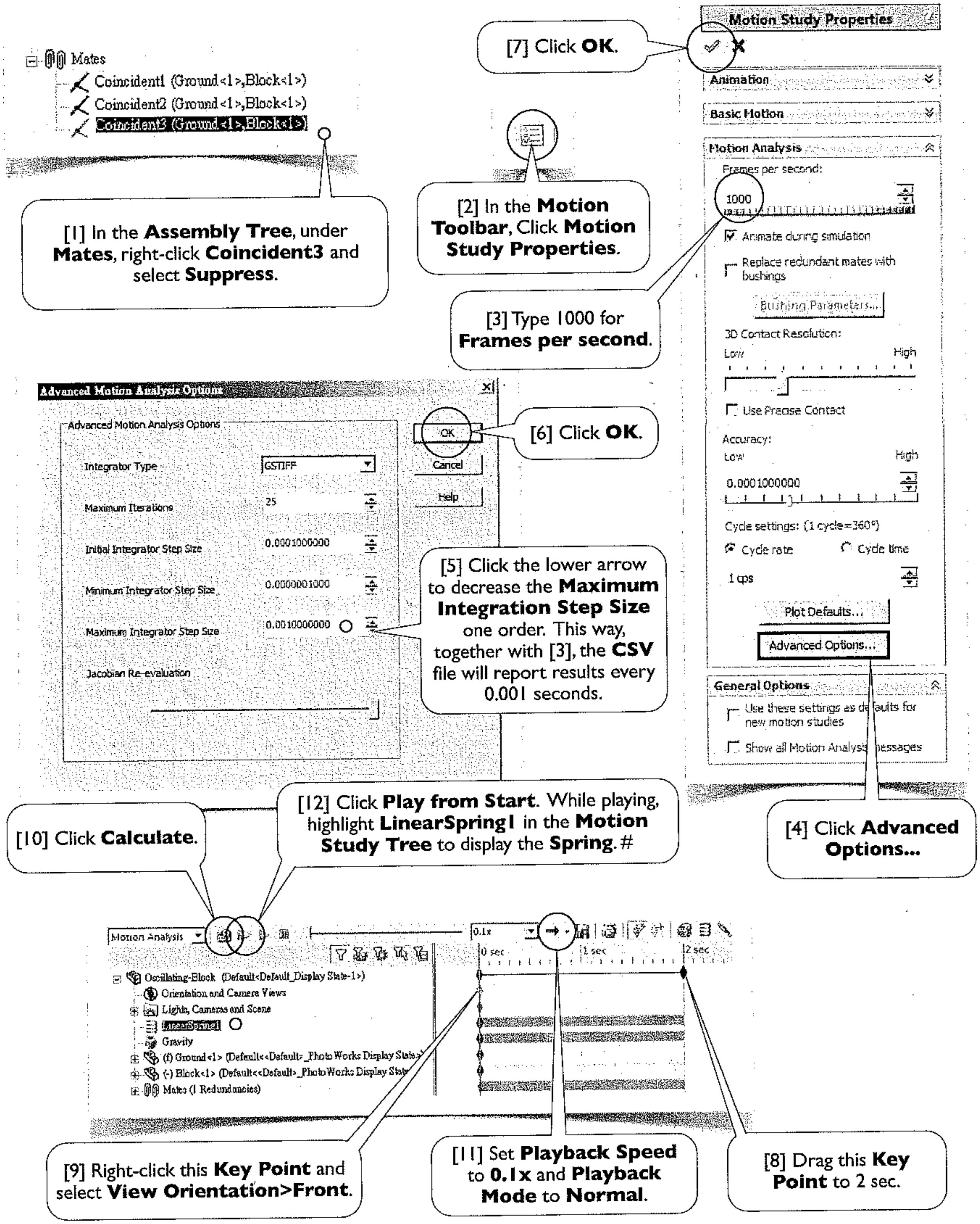
[4] Click **Specify coefficient**.

[5] Type 0.3 for **Dynamic Friction Coefficient**.

[6] **SOLIDWORKS Motion** uses a friction model for **Mates** that includes **Joint dimensions** effects. To disable the **Joint dimensions** effects, simply input large dimensions. Here, we type 500 (m) for all the three dimensions.

[7] Click **OK**. Click **OK** again to dismiss the **Property Box**. #

3.1-9 Calculate and Animate **Results**



[1] In the **Assembly Tree**, under **Mates**, right-click **Coincident3** and select **Suppress**.

[2] In the **Motion Toolbar**, Click **Motion Study Properties**.

[3] Type 1000 for **Frames per second**.

[4] Click **Advanced Options...**

[5] Click the lower arrow to decrease the **Maximum Integration Step Size** one order. This way, together with [3], the **CSV** file will report results every 0.001 seconds.

[6] Click **OK**.

[7] Click **OK**.

[8] Drag this **Key Point** to 2 sec.

[9] Right-click this **Key Point** and select **View Orientation>Front**.

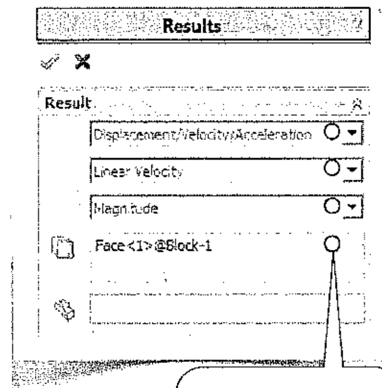
[10] Click **Calculate**.

[11] Set **Playback Speed** to **0.1x** and **Playback Mode** to **Normal**.

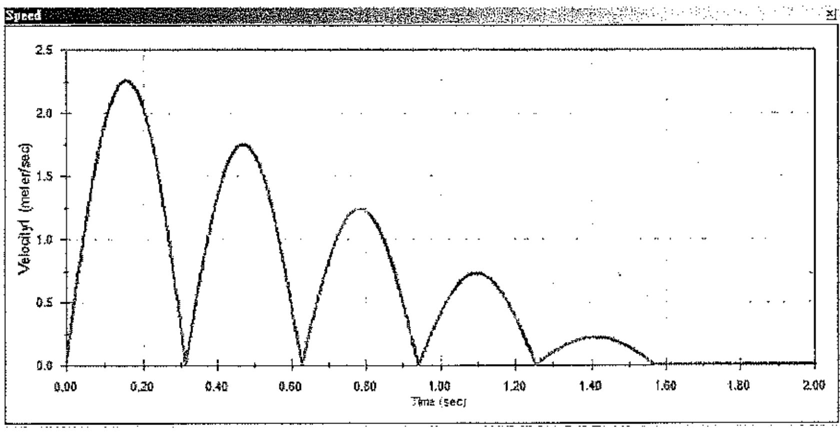
[12] Click **Play from Start**. While playing, highlight **LinearSpring1** in the **Motion Study Tree** to display the **Spring**. #

3.1-10 Results: Velocity of **Block**

[1] In **Motion Toolbar**, click **Results and Plots**.

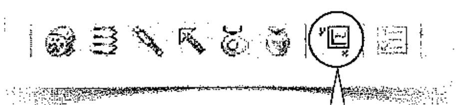


[2] Click the **Block**.

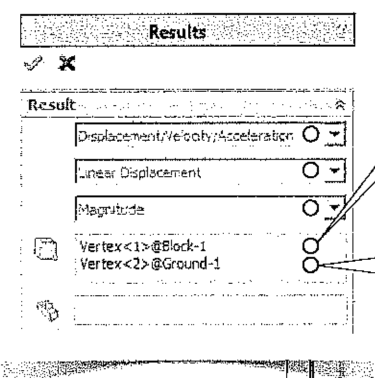


[3] Rename the plot as **Speed**. Export the file as **Speed.CSV**.#

3.1-11 Results: Displacement of **Block**

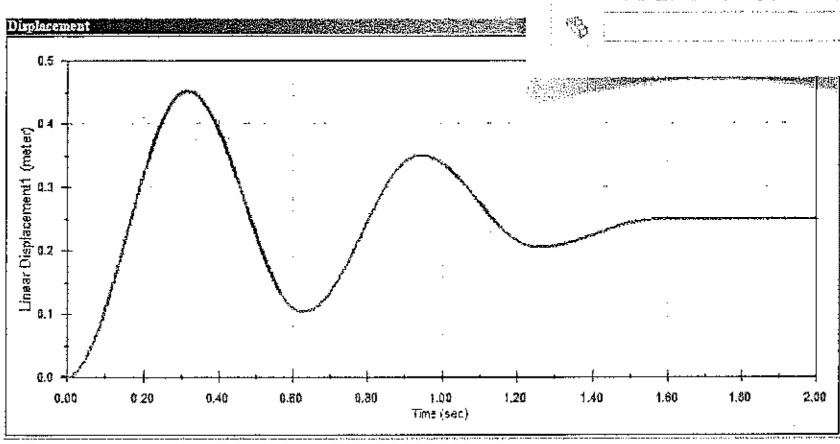


[1] In **Motion Toolbar**, click **Results and Plots**.



[2] Select the front-lower-left corner of **Block** (see 3.1-4[15], page 62).

[3] Select the front-lower-left corner of **Ground** (see 3.1-4[14], page 62).



[4] Rename the plot as **Displacement**. Export the file as **Displacement.CSV**.#

3.1-12 Results: Period of the Oscillation

[1] The angular frequency of an **undamped free vibration** can be calculated

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \text{ N/m}}{1 \text{ kg}}} = 10 \text{ rad/s}$$

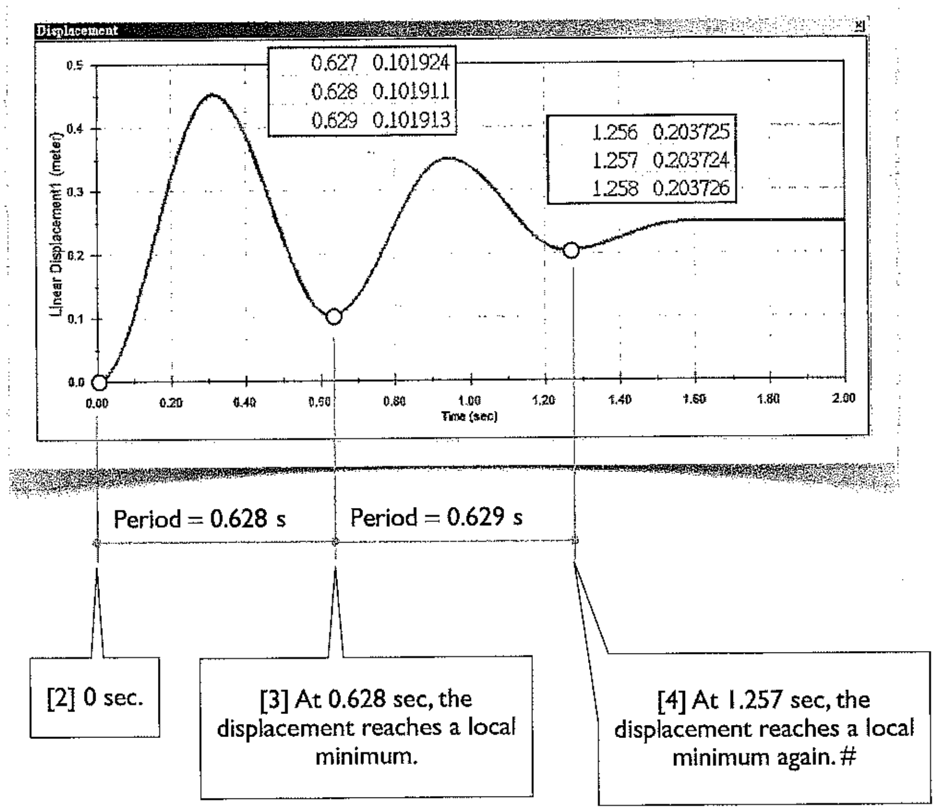
In terms of Hertz (cycle/s), the frequency is

$$f = \frac{\omega}{2\pi} = \frac{10 \text{ rad/s}}{2\pi \text{ rad}} = 1.59155 \text{ cycle/s (Hz)}$$

The period is then

$$p = \frac{1}{f} = \frac{1}{1.592 \text{ cycle/s}} = 0.62832 \text{ s/cycle} \qquad (1)$$

The period in our case [2-4] is close to the value calculated in Eq. (1), although our case is a **damped free vibration** case (due to the friction).



3.1-13 Conservation of Energy

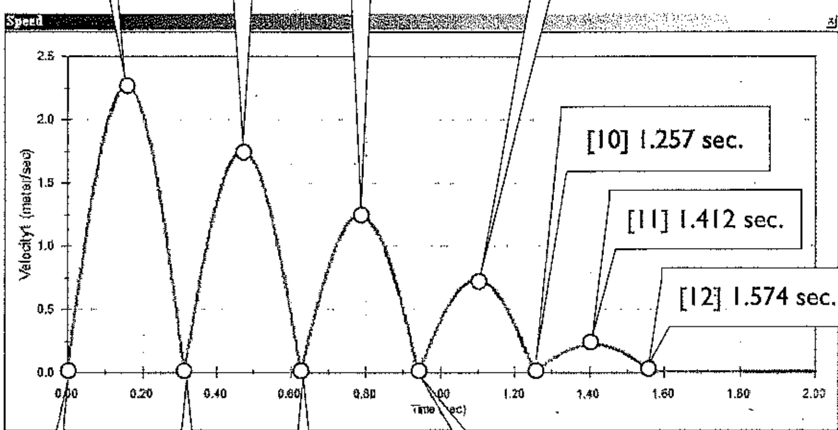
[1] Here, we illustrate Eq. 3.1-1(2) (page 59) at some selected time points [2-12], when the speed of the **Block** either reaches a maximum or approaches zero. These time points are obtained by looking up **Speed.CSV**. The results [13-20] show that the total energy (including the dissipated energy due to friction) is conserved at these time points. In fact, the total energy is conserved at any time, not just these selected time points. We select these points simply because the calculation of travel distance is easier.

[3] 0.157 sec.

[5] 0.471 sec.

[7] 0.785 sec.

[9] 1.099 sec.



[10] 1.257 sec.

[11] 1.412 sec.

[12] 1.574 sec.

[2] 0 sec.

[4] 0.314 sec.

[6] 0.628 sec.

[8] 0.943 sec.

[18] $V_g = mgD(\sin 30^\circ) = (1)(9.80655)D(\sin 30^\circ)$.

[13] The speed is copied from **Speed.CSV**.

[15] This is the accumulative travel distance. For example, at $t = 0.471$ s, $0.450969 + |0.276684 - 0.450969| = 0.625254$.

[19] $V_e = k(0.3 - D)^2/2 = (100)(0.3 - D)^2/2$.

Time t (s)	Speed v (m/s)	Displacement D (m)	Travel Distance S (m)	Work Done by Friction $U_{0\to t}$ (J)	Kinetic Energy T (J)	Gravitational Potential Energy V_g (J)	Elastic Potential Energy V_e (J)	Total Energy (J)
0	0	0	0	0	0	0	4.500000	4.500
0.157	2.254733	0.225364	0.225364	-0.574193	2.541910	1.105035	0.278523	4.500
0.314	0.003991	0.450969	0.450969	-1.148998	0.000008	2.211248	1.139585	4.500
0.471	1.745399	0.276684	0.625254	-1.593049	1.523209	1.356672	0.027181	4.500
0.628	0.004531	0.101911	0.800028	-2.038344	0.000010	0.499701	1.961968	4.500
0.785	1.235857	0.225249	0.923365	-2.352590	0.763671	1.104467	0.279388	4.500
0.943	0.004576	0.349091	1.047208	-2.668121	0.000010	1.711708	0.120498	4.500
1.099	0.726886	0.276546	1.119753	-2.852955	0.264182	1.355996	0.027504	4.501
1.257	0.001167	0.203724	1.192576	-3.038495	0.000001	0.998923	0.463457	4.501
1.412	0.217506	0.225631	1.214483	-3.094312	0.023654	1.106343	0.276537	4.501
1.574	0.000235	0.247236	1.236088	-3.149359	0.000000	1.212280	0.139200	4.501

[14] The displacement is copied from **Displacement.CSV**.

[16] $U_{0\to t} = (\mu mg \cos 30^\circ)S = 0.3(1)(9.80665)(\cos 30^\circ)S$.

[17] $T = mv^2/2 = (1)v^2/2$.

[20] Total energy $= T + V_g + V_e - U_{0\to t}$.#

3.1-14 Principle of Minimum Potential Energy

[1] The 2nd, 3rd, and 4th columns of this table are copied from 3.1-13[14, 18, 19], last page.

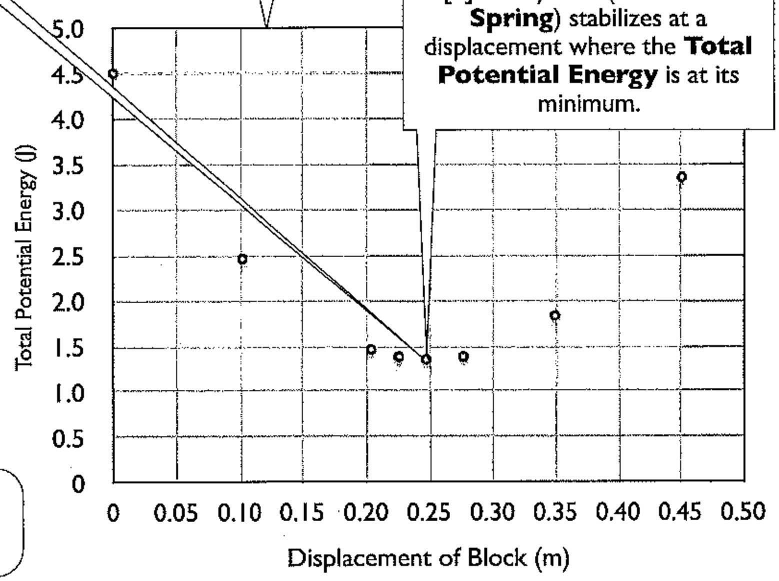
[2] Total potential energy $V = V_g + V_e$.

Time t (s)	Displacement D (m)	Gravitational Potential Energy V_g (J)	Elastic Potential Energy V_e (J)	Total Potential Energy $V_g + V_e$ (J)
0	0	0	4.500000	4.5000
0.157	0.225364	1.105035	0.278523	1.3836
0.314	0.450969	2.211248	1.139585	3.3508
0.471	0.276684	1.356672	0.027181	1.3839
0.628	0.101911	0.499701	1.961968	2.4617
0.785	0.225249	1.104467	0.279388	1.3839
0.943	0.349091	1.711708	0.120498	1.8322
1.099	0.276546	1.355996	0.027504	1.3835
1.257	0.203724	0.998923	0.463457	1.4624
1.412	0.225631	1.106343	0.276537	1.3829
1.574	0.247236	1.212280	0.139200	1.3515

[6] Any infinitesimal change of displacement will not change the total potential energy; therefore doing zero work.

[3] This is a plot of the **Total Potential Energy** versus the **Displacement** D.

[4] The system (**Block** and **Spring**) stabilizes at a displacement where the **Total Potential Energy** is at its minimum.



[5] The **principle of minimum potential energy** states that, *among all possible configurations, a structural system stabilizes at a configuration in which the total potential energy is at its minimum*. The **Principle of virtual work** is actually another way of saying the principle of minimum potential energy: *when a structural system is in a stable configuration (i.e., the total potential energy is at its minimum), any infinitesimal changes of the configuration will not change the total potential energy; therefore doing zero work* [6].

Wrap Up

[7] Save all files and exit **SOLIDWORKS**. #

3.1-15 (Do It Yourself) Conservation of Energy: Block and Wedge

Revisit the **Block and Wedge** (Section 2.1) and investigate the energy at some arbitrarily selected time points. Verify the **conservation of energy**

$$T_0 + V_0 = T_t + V_t \quad (1)$$

where T_0 and V_0 are respectively the initial ($t = 0$) kinetic energy and potential energy of the system, and T_t and V_t are respectively the kinetic energy and potential energy of the system at time t. The table below demonstrates a possible results.

Time (s)	Block Kinetic Energy T_1 (J)	Wedge Kinetic Energy T_2 (J)	Block Y-Displacement (m)	Block Potential Energy V (J)	Total Energy $T_1 + T_2 + V$ (J)
0	0	0	0	0	0
0.1	0.739152	0.178791	-0.015601	-0.917946	-0.000003
0.2	2.956610	0.715165	-0.062403	-3.671777	-0.000003
0.3	6.652371	1.609122	-0.140406	-8.261496	-0.000003
0.4	11.826438	2.860661	-0.249611	-14.687101	-0.000003