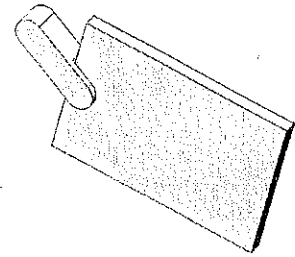


Section 6.1

Newton's 2nd Law: Double Pendulum



6.1-1 Introduction

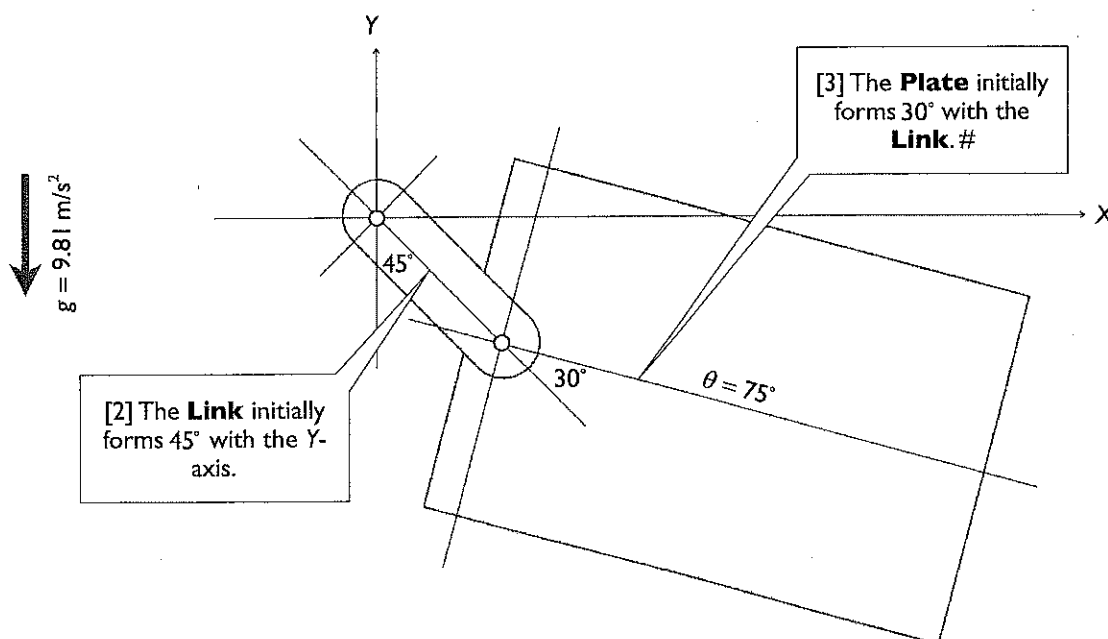
[1] Consider a **Double Pendulum** consisting of a **Link** and a **Plate** [2, 3]. The system is released from an initial configuration in which the **Link** forms 45° with a vertical plane and the **Plate** forms 30° with the **Link**.

In this section, we'll perform a simulation for this system and demonstrate how the dynamic behavior is governed by Newton's 2nd Law:

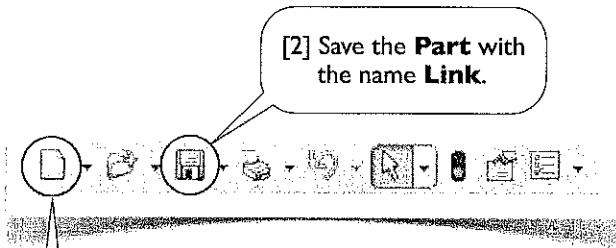
$$\sum F_x = m\bar{a}_x, \quad \sum F_y = m\bar{a}_y, \quad \sum M_G = \bar{I}\alpha \quad (1)$$

where XY -plane is the motion plane, m is the mass of the body, \bar{a}_x and \bar{a}_y are the acceleration components at the mass center, α is the angular acceleration of the body, and \bar{I} is the moment of inertia with respect to an axis perpendicular to the motion plane and passing through the mass center G . The SI unit for \bar{I} is $\text{kg}\cdot\text{m}^2$.

The geometry details of the **Link** and the **Plate** (including the locations of mass center and the moment of inertia \bar{I}) are not shown in this page, but will be illustrated later.



6.1-2 Start Up and Create a Part: **Link**

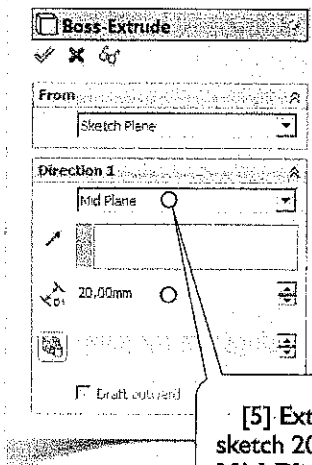
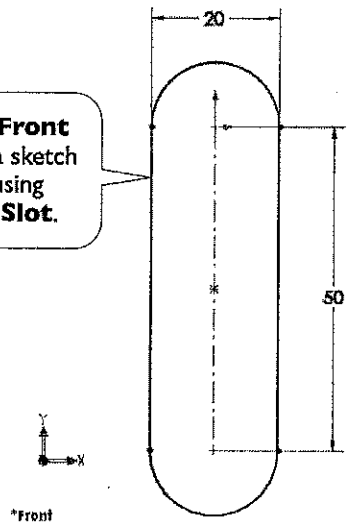


[1] Launch **SOLIDWORKS** and click **New** to create a new **Part**.

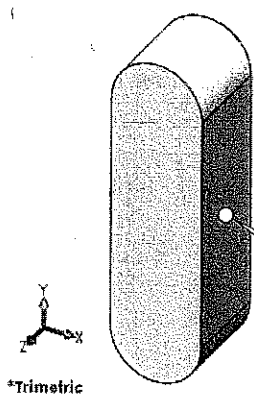
[2] Save the **Part** with the name **Link**.

[3] Set up **MMGS** unit system with two decimal places.

[4] On the **Front** plane, draw a sketch like this, using **Straight Slot**.

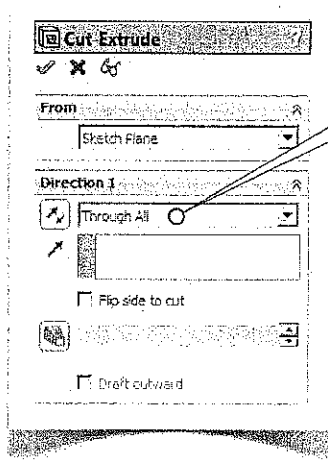
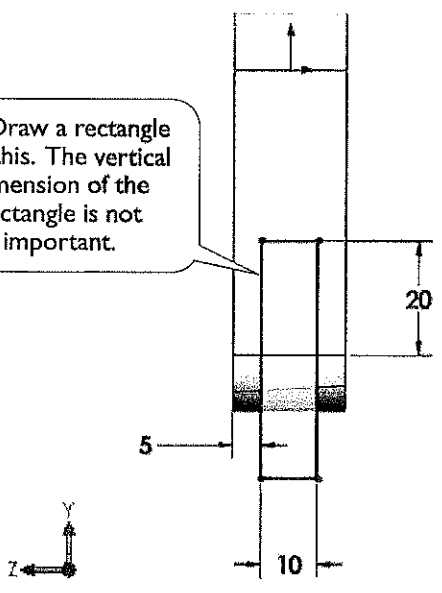


[5] Extrude the sketch 20 mm, using **Mid Plane** option.

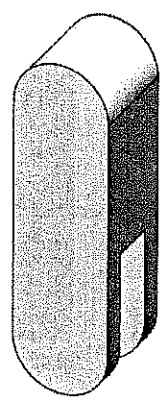


[6] Right-click this side face and select **Sketch**.

[7] Draw a rectangle like this. The vertical dimension of the rectangle is not important.



[8] **Extruded Cut**, using **Through All** option. #



6.1-3 Create a Part: **Plate**

[1] Click **New** to create a new **Part**.

[2] Save the **Part** with the name **Plate**.

[3] Set up **MMGS** unit system with two decimal places for **Length** in **Basic Units** and 5 decimal places for **Length** in **Mass/Section Properties**.

[4] On the **Front** plane, draw a sketch like this.

[5] Extrude the sketch 10 mm, using **Mid Plane** option.

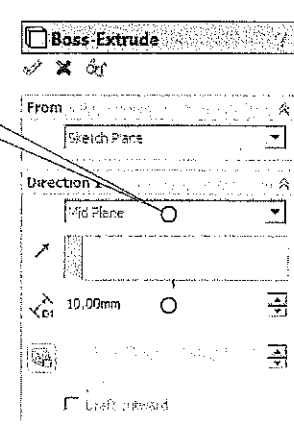
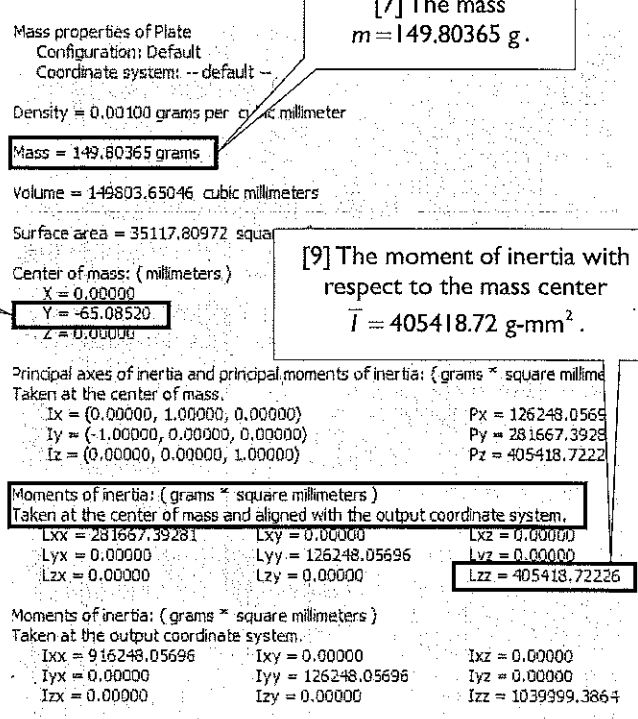
[6] In the **Evaluate Toolbar**, click **Mass Properties**.

[7] The mass $m = 149.80365 \text{ g}$.

[8] The mass center locates at $Y = -65.08520 \text{ mm}$.

[9] The moment of inertia with respect to the mass center $\bar{I} = 405418.72 \text{ g}\cdot\text{mm}^2$.

[10] Close **Mass Properties**. #

Mass properties of Plate
 Configuration: Default
 Coordinate system: -- default --
 Density = 0.00100 grams per cubic millimeter
Mass = 149.80365 grams
 Volume = 149803.65046 cubic millimeters
 Surface area = 35117.80972 square millimeters
 Center of mass: (millimeters)
 X = 0.00000
Y = -65.08520
 Z = 0.00000

Principal axes of inertia and principal moments of inertia: (grams * square millimeter)
 Taken at the center of mass.
 Ix = (0.00000, 1.00000, 0.00000) Px = 126248.05696
 Iy = (-1.00000, 0.00000, 0.00000) Py = 281667.39281
 Iz = (0.00000, 0.00000, 1.00000) Pz = 405418.72226

Moments of inertia: (grams * square millimeters)
 Taken at the center of mass and aligned with the output coordinate system.
 Lxx = 281667.39281 Lxy = 0.00000 Lxz = 0.00000
 Lyx = 0.00000 Lyy = 126248.05696 Lyz = 0.00000
 Lzx = 0.00000 Lzy = 0.00000 **Lzz = 405418.72226**

Moments of inertia: (grams * square millimeters)
 Taken at the output coordinate system.
 Ixx = 916248.05696 Ixy = 0.00000 Ixz = 0.00000
 Iyx = 0.00000 Iyy = 126248.05696 Iyz = 0.00000
 Izx = 0.00000 Izy = 0.00000 Izz = 1039999.3864

*Front

*Trimetric

6.1-4 Create an Assembly: **Pendulum**

[1] Click **New** and create an **Assembly**.

[2] In the **Head-Up Toolbar**, turn on **View Origins**.

[3] In the **Begin Assembly Property Box**, select **Link** and click the **Origin** of the **Assembly**.

[4] Save the **Assembly** with the name **Pendulum**.

[5] Set up **MMGS** unit system with two decimal places.

[6] In the **Assembly Toolbar**, click **Insert Components** and park **Plate** anywhere.

[7] Right-click **Link<1>** and select **Float**.

[8] In **Assembly Toolbar**, click **Mate**.

*Trimetric

- (f) Link<1> (Default<<Default>_Photo Works Display State>)
- (-) Plate<1> (Default<<Default>_Photo Works Display State>)

[9] We'll create these 6 **Mates** in [10-22].

[10] The **Plate** and the **Link** are connected with a revolute joint (see [11, 12]).

[13] The **Link's** inner frontal face [14] and the **Plate's** backside face [15] are **Coincident**.

[16] The **Links's** and the **Assembly's Origins** are **Coincident**. Uncheck **Align axes** (see [17]) before clicking **OK**.

[18] The **Link's** and the **Assembly's Front** planes are **Coincident**.

[20] The **Assembly's** and the **Link's Right** planes form an **Angle** of 45 degrees. Click **Flip dimension** if necessary. This is the initial configuration of the **Link**. We'll suppress this **Mate** later.

[21] The **Link's** and the **Plate's Right** planes form an **Angle** of 30 degrees. Click **Flip dimension** if necessary. This is the initial configuration of the **Plate**. We'll suppress this **Mate** later.

[11] **Concentric1** is created by clicking this cylindrical face...

[15] The **Plate's** backside face.

[14] The **Link's** inner frontal face.

[12] And this cylindrical face.

[17] Uncheck **Align axes**.

[19] Now, use your mouse to test the assembly and make sure all the **Mates** are correct.

[22] This is the initial configuration. Dismiss **Mate** box. Save the file. #

The image shows a CAD software interface with a 'Mates' list on the left. The list contains six entries: Concentric1 (Link<1>,Plate<1>), Coincident1 (Link<1>,Plate<1>), Coincident2 (Link<1>,Origin), Coincident3 (Link<1>,Front), Angle1 (Right,Link<1>), and Angle2 (Link<1>,Plate<1>). The 'Align axes' checkbox is unchecked. The main area shows a 3D assembly of a plate and a link. Callouts [11] through [22] provide instructions for creating and configuring these mates. Callout [11] points to a cylindrical face on the link. Callout [12] points to another cylindrical face. Callout [14] points to the inner frontal face of the link. Callout [15] points to the backside face of the plate. Callout [17] points to the 'Align axes' checkbox in the software interface. Callout [19] points to the assembly. Callout [22] points to the assembly in a different configuration. The software interface also shows 'Trimetric' coordinate systems for the link and plate.

6.1-5 Create a **Motion Study** and Set Up **Gravity**

[1] Click **Motion Study 1** tab.

[2] Select **Motion Analysis**.

[3] In **Motion Toolbar**, click **Gravity**.

[4] Click **Y**.

[5] Click **OK**. #

6.1-6 Calculate and Animate **Results**

[1] In the **Assembly Tree**, under **Mates**, suppress **Angle1** and **Angle2** (see 6.1-4[20, 21], last page).

[2] In the **Motion Toolbar**, Click **Motion Study Properties** and type 1000 for **Frames per second**.

[3] Drag this **Key Point** to **1 sec**.

[4] Right-click this **Key Point** and select **View Orientation>Front**.

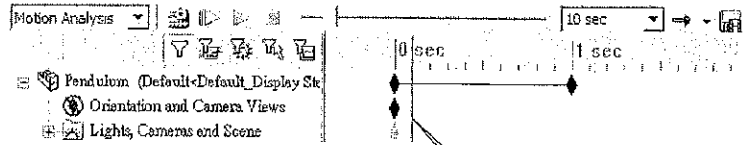
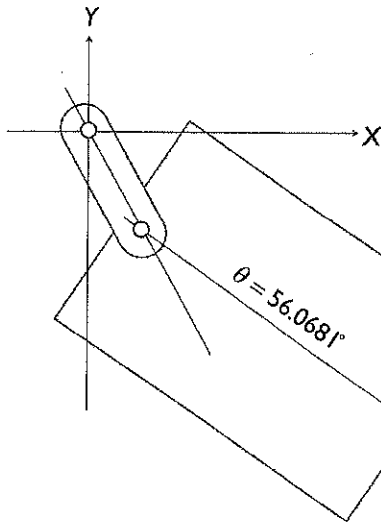
[5] Click **Calculate**.

[6] Set **Playback Speed** to **10 sec**.

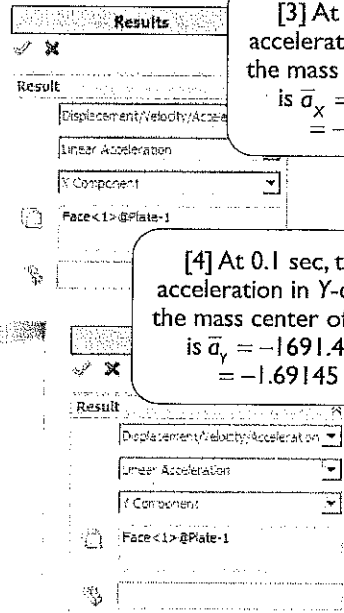
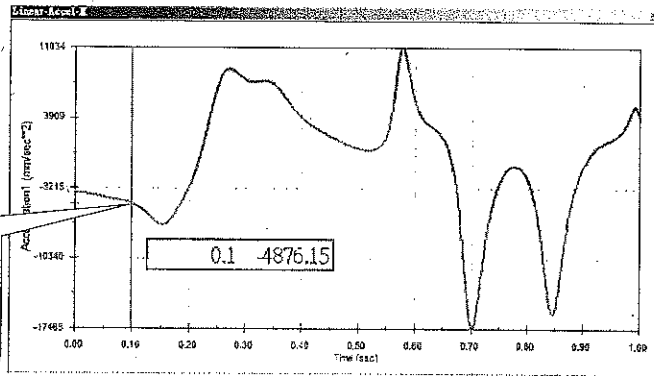
[7] Select **Normal** for **Playback Mode**.

[8] Click **Play from Start**. #

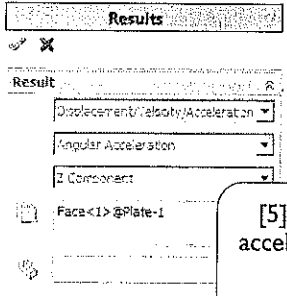
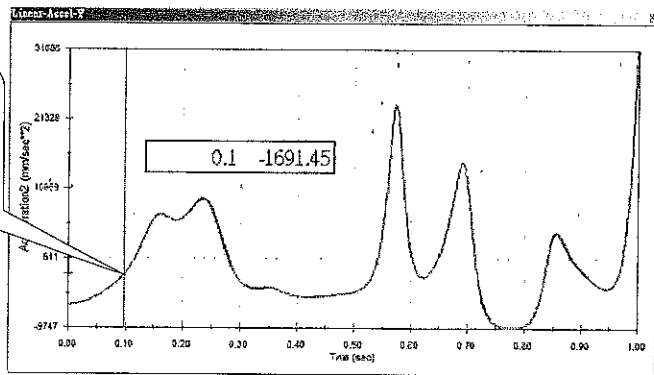
6.1-7 Results: Linear and Angular Accelerations of the **Plate**



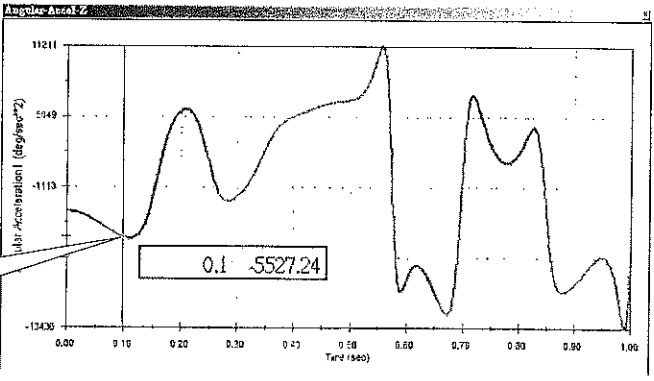
[2] At $t = 0.1$ sec, the **Plate** forms an angle $\theta = 56.068^\circ$ with the Y-axis (see 6.1-8[1], next page)



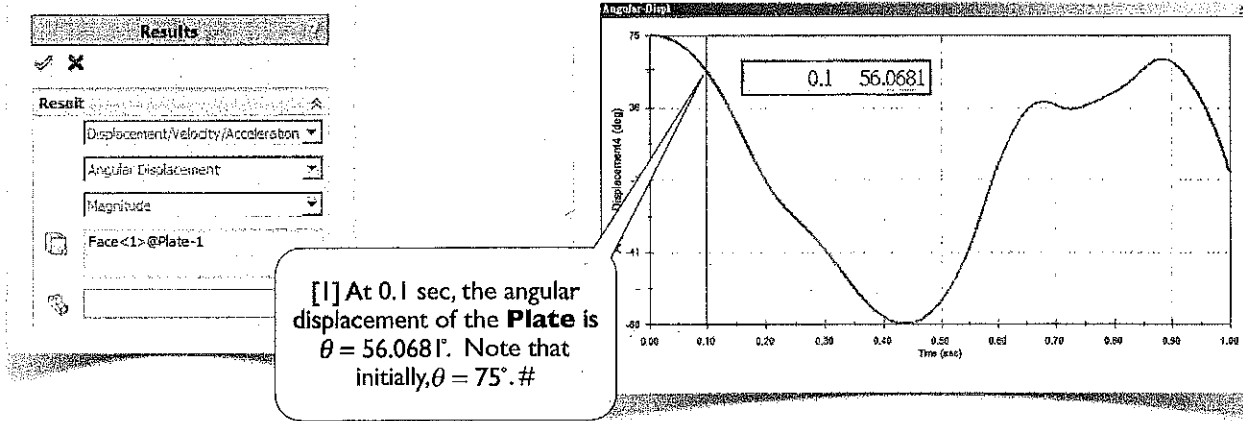
[4] At 0.1 sec, the linear acceleration in Y-direction at the mass center of the **Plate** is $\bar{a}_y = -1691.45 \text{ mm/s}^2 = -1.69145 \text{ m/s}^2$.



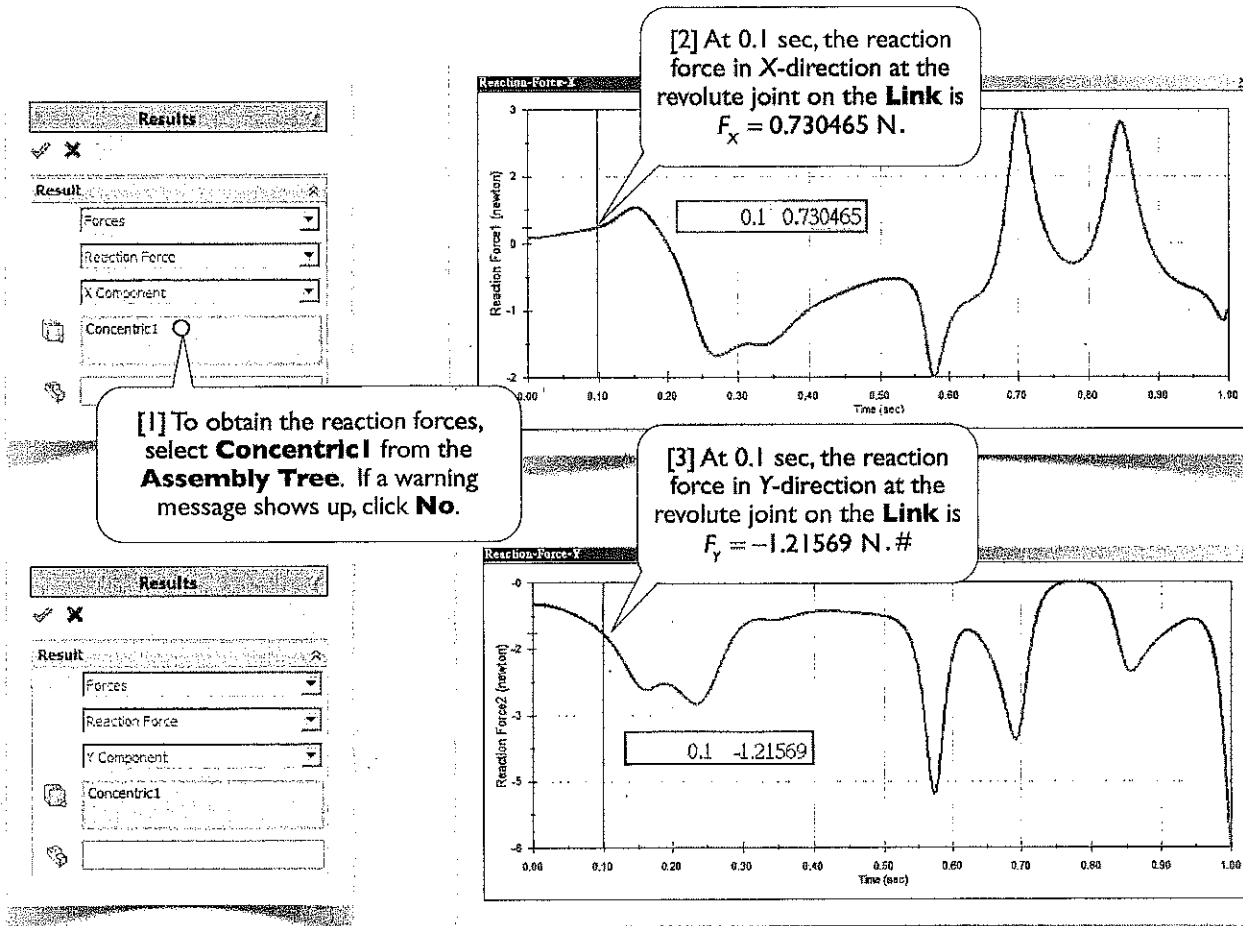
[5] At 0.1 sec, the angular acceleration in Z-direction of the **Plate** is $\alpha = -5527.24 \text{ deg/s}^2 = -96.4685 \text{ rad/s}^2$.



6.1-8 Results: Angular Displacement of the **Plate**



6.1-9 Results: Reaction Forces at **Concentric1**



6.1-10 Newton's 2nd Law: **Plate**

[1] Newton's 2nd Law for a rigid body in plane motion (Eq. 6.1-1(1), page 122) states that the external forces and moments acting on a rigid body are equivalent to the **effective force and moments** acting on the particle. The effective force of a rigid body is simply the product of its mass and the acceleration at the mass center; the effective moment of a rigid body is the product of its moment of inertia \bar{I} and the angular acceleration α .

The external forces acting on the **Plate** and the effective forces and moment on the **Plate** are shown in [2-8]. It's easy to confirm that these forces and moments indeed satisfy Newton's 2nd Law; i.e., in X-direction,

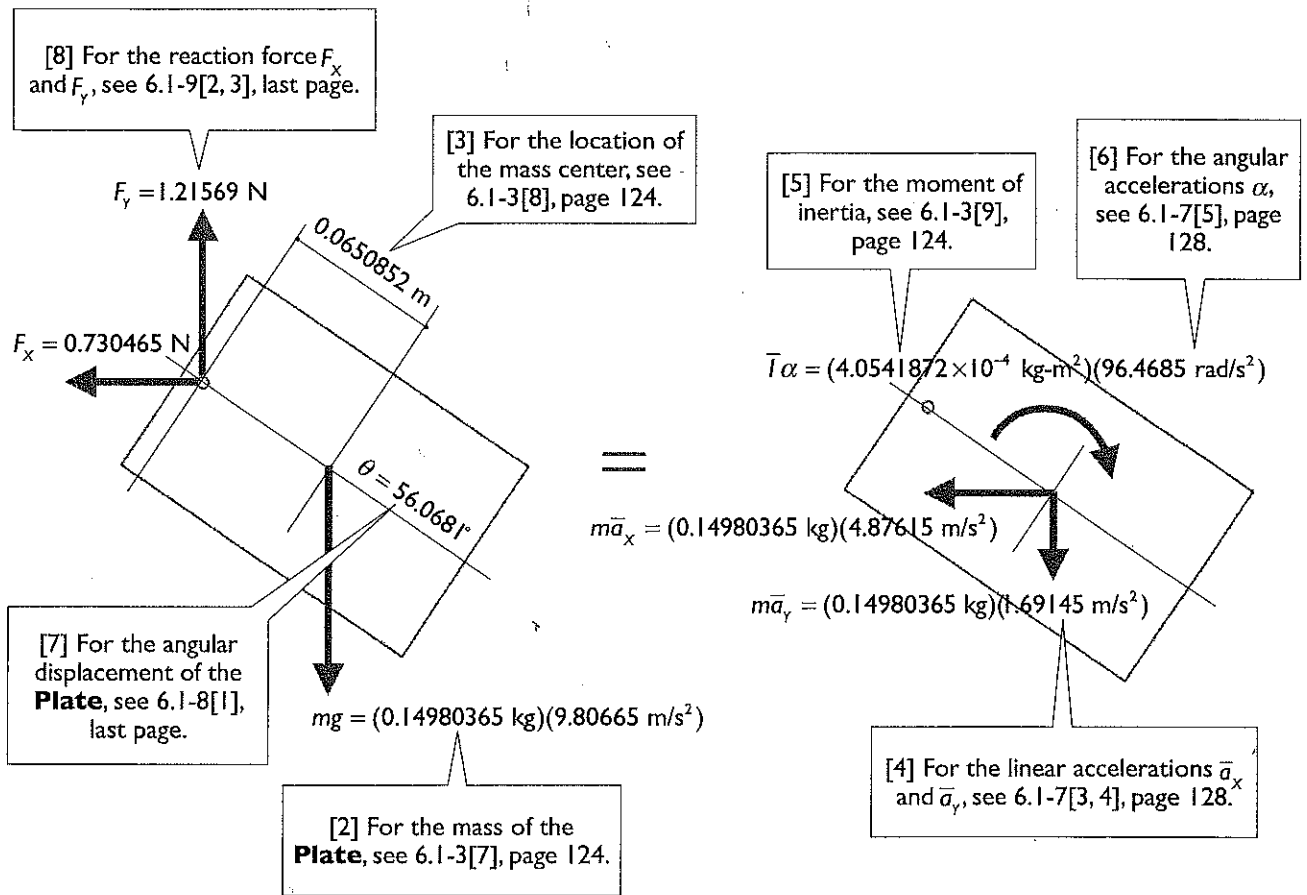
$$-0.730465 \approx (0.14980365)(-4.87615)$$

In Y-direction,

$$1.21569 - (0.14980365)(9.80665) \approx (0.14980365)(-1.69145)$$

Taking the moment about the mass center, we have

$$(0.730465)(0.0650852 \cos 56.0681^\circ) - (1.21569)(0.0650852 \sin 56.0681^\circ) \approx -(4.0541872 \times 10^{-4})(96.4685)$$



Wrap Up

[9] Save all files and exit **SOLIDWORKS**. #