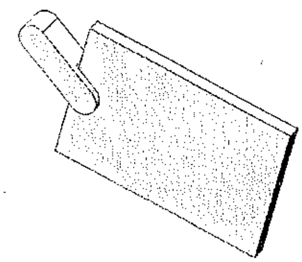
Section 6.1

Newton's 2nd Law: Double Pendulum



6.1-1 Introduction

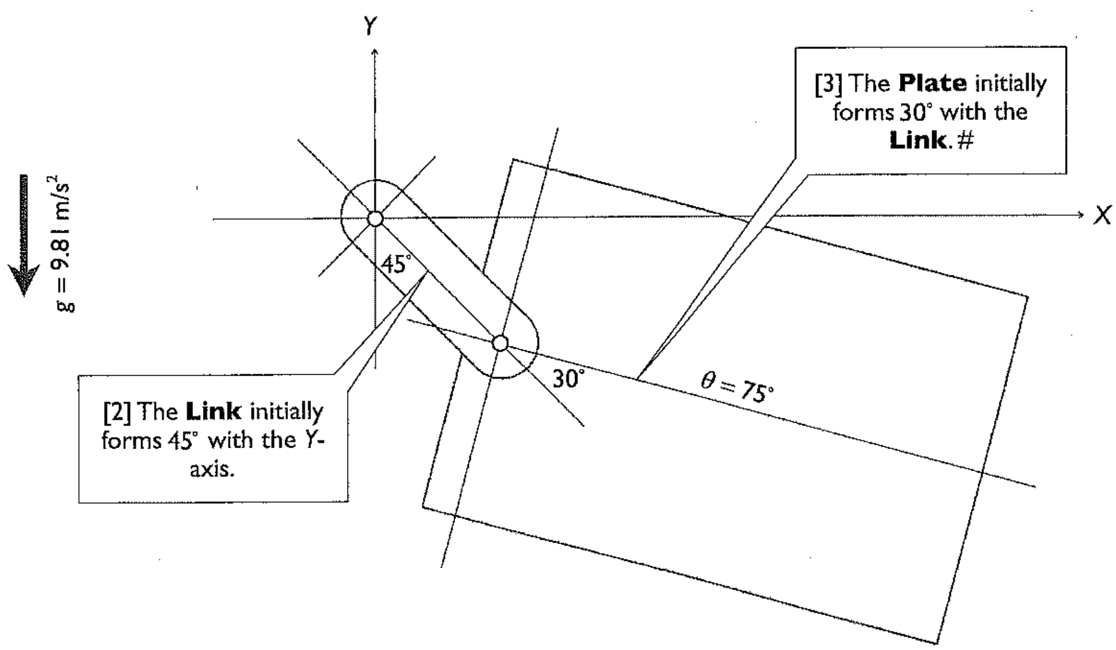
[1] Consider a **Double Pendulum** consisting of a **Link** and a **Plate** [2, 3]. The system is released from an initial configuration in which the **Link** forms 45° with a vertical plane and the **Plate** forms 30° with the **Link**.

In this section, we'll perform a simulation for this system and demonstrate how the dynamic behavior is governed by Newton's 2nd Law:

$$\sum F_X = m\bar{a}_X, \ \sum F_Y = m\bar{a}_Y, \ \sum M_G = \bar{I}\alpha \qquad (1)$$

where XY-plane is the motion plane, m is the mass of the body, $\bar{a}_X$ and $\bar{a}_Y$ are the acceleration components at the mass center, α is the angular acceleration of the body, and $\bar{I}$ is the moment of inertia with respect to an axis perpendicular to the motion plane and passing through the mass center G. The SI unit for $\bar{I}$ is kg-m^2.

The geometry details of the **Link** and the **Plate** (including the locations of mass center and the moment of inertia $\bar{I}$) are not shown in this page, but will be illustrated later.

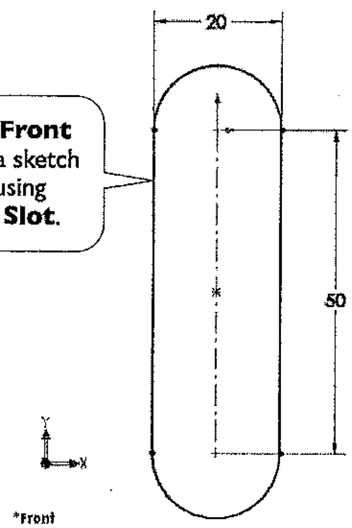


6.1-2 Start Up and Create a Part: **Link**

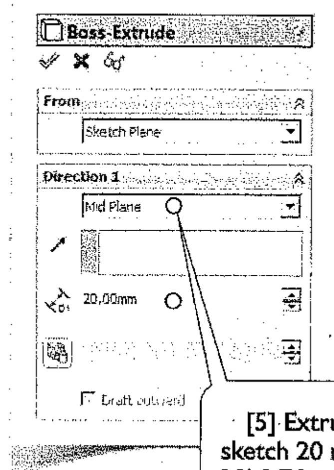
[1] Launch **SOLIDWORKS** and click **New** to create a new **Part**.

[2] Save the **Part** with the name **Link**.

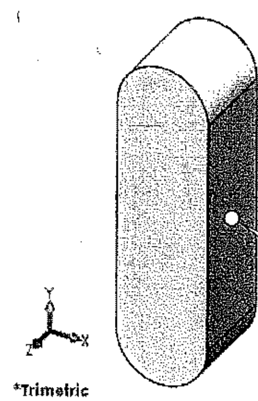
[3] Set up **MMGS** unit system with two decimal places.



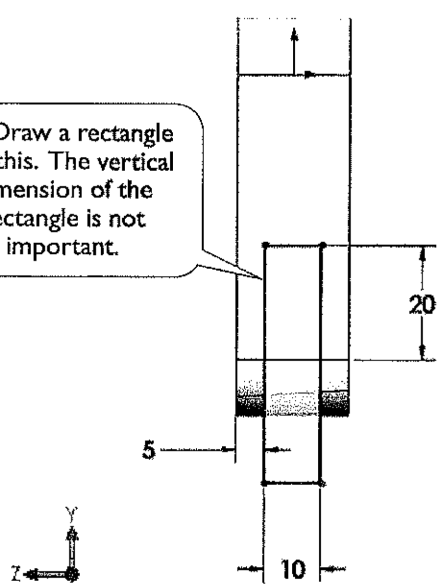
[4] On the **Front** plane, draw a sketch like this, using **Straight Slot**.



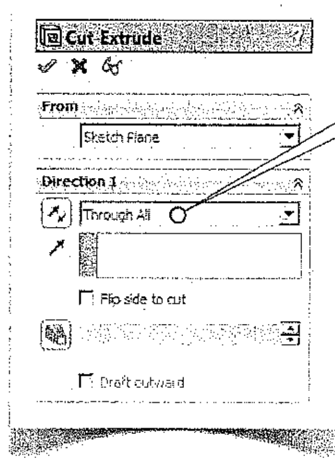
[5] Extrude the sketch 20 mm, using **Mid Plane** option.



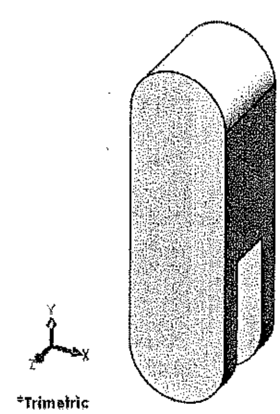
[6] Right-click this side face and select **Sketch**.



[7] Draw a rectangle like this. The vertical dimension of the rectangle is not important.



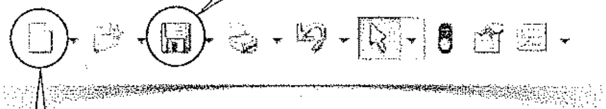
[8] **Extruded Cut**, using **Through All** option. #



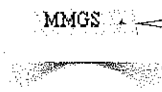
6.1-3 Create a Part: **Plate**

[1] Click **New** to create a new **Part**.

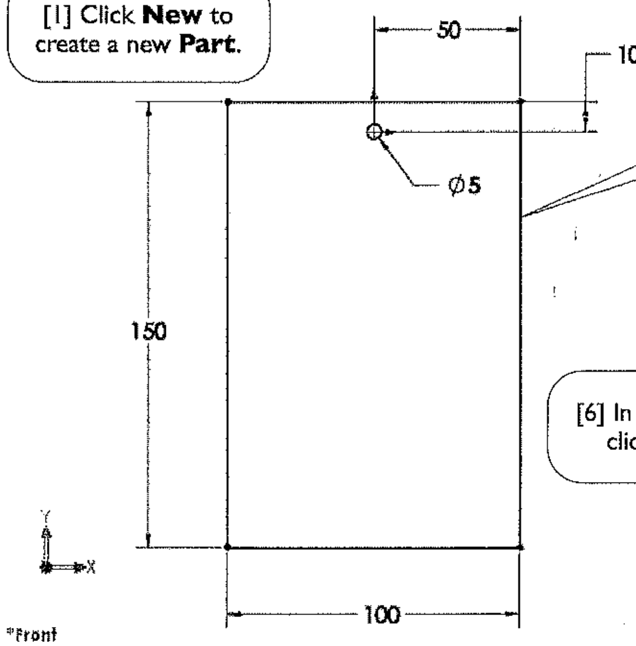
[2] Save the **Part** with the name **Plate**.



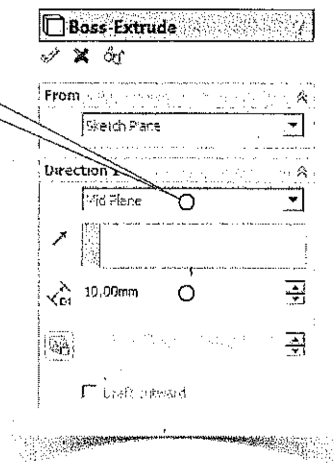
[3] Set up **MMGS** unit system with two decimal places for **Length** in **Basic Units** and 5 decimal places for **Length** in **Mass/Section Properties.**



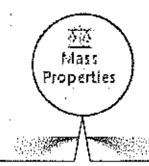
[4] On the **Front** plane, draw a sketch like this.

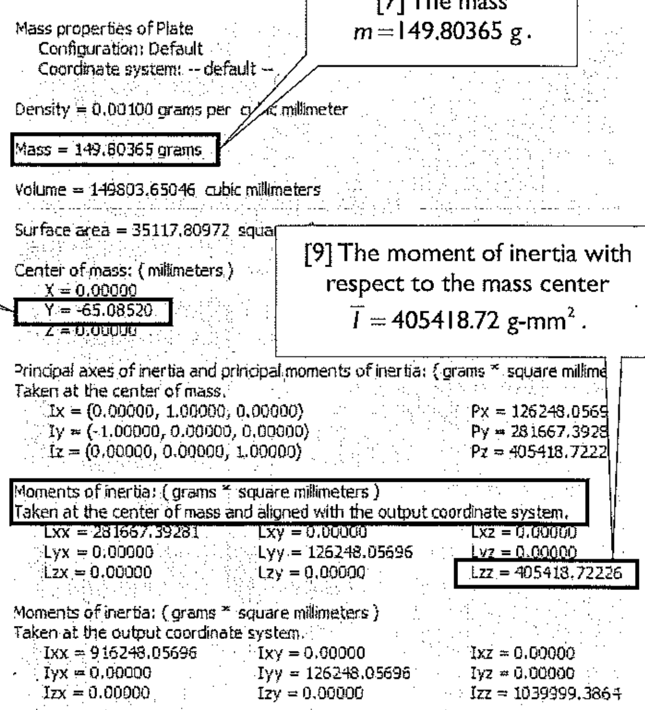


[5] Extrude the sketch 10 mm, using **Mid Plane** option.



[6] In the **Evaluate Toolbar**, click **Mass Properties**.




Mass properties of Plate
Configuration: Default
Coordinate system: -- default --

Density = 0.00100 grams per cubic millimeter

Mass = 149.80365 grams

Volume = 149803.65046 cubic millimeters

Surface area = 35117.80972 square millimeters

Center of mass: (millimeters)
X = 0.00000
Y = -65.08520
Z = 0.00000

Principal axes of inertia and principal moments of inertia: (grams * square millimeters)
Taken at the center of mass.
Ix = (0.00000, 1.00000, 0.00000) Px = 126248.0569
Iy = (-1.00000, 0.00000, 0.00000) Py = 281667.3928
Iz = (0.00000, 0.00000, 1.00000) Pz = 405418.7222

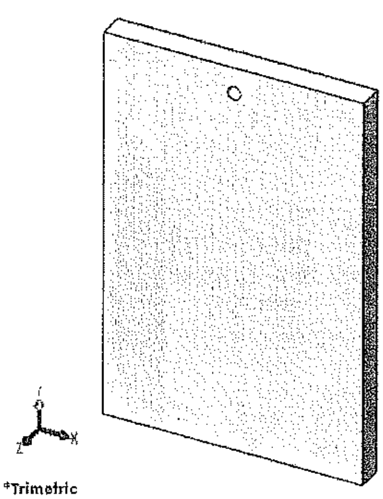
Moments of inertia: (grams * square millimeters)
Taken at the center of mass and aligned with the output coordinate system.
Lxx = 281667.39281 Lxy = 0.00000 Lxz = 0.00000
Lyx = 0.00000 Lyy = 126248.05696 Lyz = 0.00000
Lzx = 0.00000 Lzy = 0.00000 Lzz = 405418.72226

Moments of inertia: (grams * square millimeters)
Taken at the output coordinate system.
Ixx = 916248.05696 Ixy = 0.00000 Ixz = 0.00000
Iyx = 0.00000 Iyy = 126248.05696 Iyz = 0.00000
Izx = 0.00000 Izy = 0.00000 Izz = 1039999.3864

[7] The mass $m = 149.80365$ g.

[8] The mass center locates at $Y = -65.08520$ mm.

[9] The moment of inertia with respect to the mass center $\bar{I} = 405418.72$ g-mm^2.

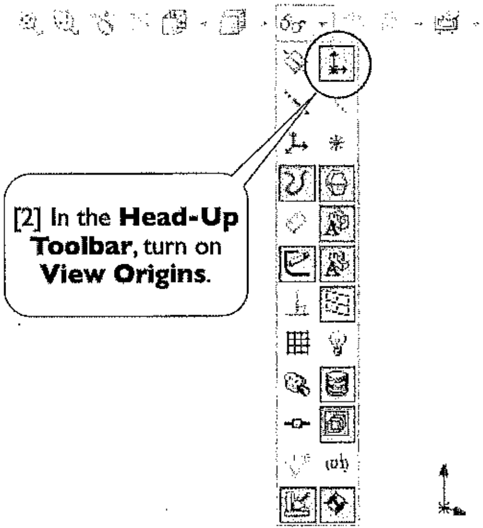


[10] Close **Mass Properties**. #

6.1-4 Create an Assembly: **Pendulum**

[1] Click **New** and create an **Assembly**.

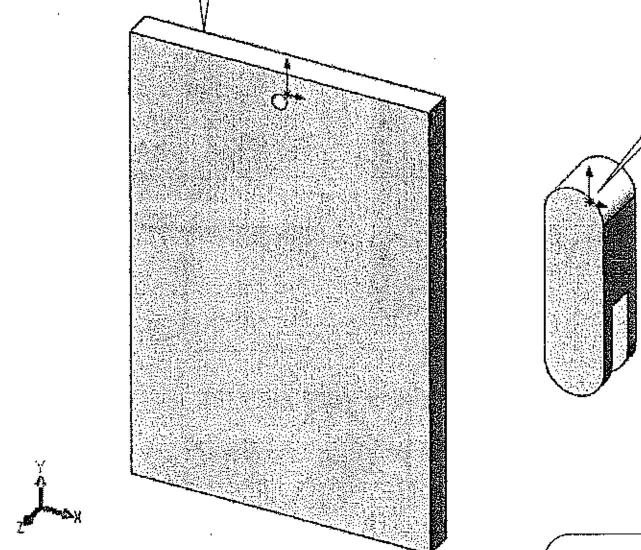
[4] Save the **Assembly** with the name **Pendulum**.



[2] In the **Head-Up Toolbar**, turn on **View Origins**.

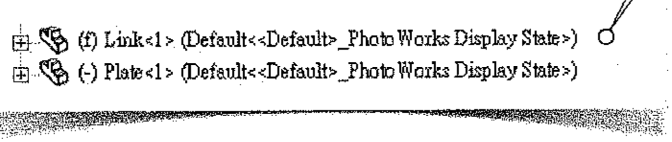
[5] Set up **MMGS** unit system with two decimal places.

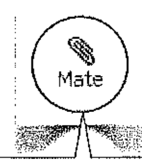
[6] In the **Assembly Toolbar**, click **Insert Components** and park **Plate** anywhere.



[3] In the **Begin Assembly Property Box**, select **Link** and click the **Origin** of the **Assembly**.

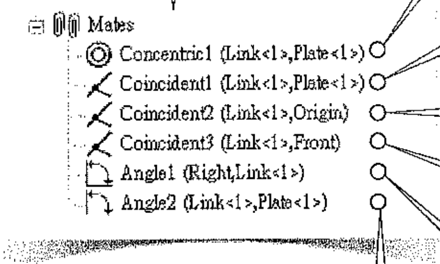
[7] Right-click **Link<1>** and select **Float**.





[8] In **Assembly Toolbar**, click **Mate**.

[9] We'll create these 6 **Mates** in [10-22].



[10] The **Plate** and the **Link** are connected with a revolute joint (see [11, 12]).

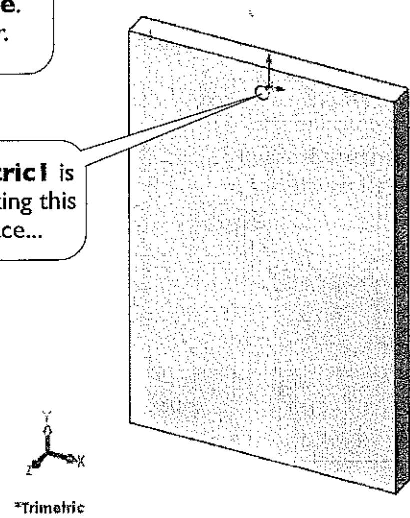
[13] The **Link's** inner frontal face [14] and the **Plate's** backside face [15] are **Coincident**.

[16] The **Links's** and the **Assembly's Origins** are **Coincident**. Uncheck **Align axes** (see [17]) before clicking **OK**.

[18] The **Link's** and the **Assembly's Front** planes are **Coincident**.

[20] The **Assembly's** and the **Link's Right** planes form an **Angle** of 45 degrees. Click **Flip dimension** if necessary. This is the initial configuration of the **Link**. We'll suppress this **Mate** later.

[21] The **Link's** and the **Plate's Right** planes form form an **Angle** of 30 degrees. Click **Flip dimension** if necessary. This is the initial configuration of the **Plate**. We'll suppress this **Mate** later.

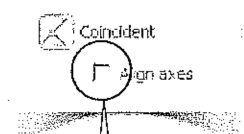


[15] The **Plate's** backside face.

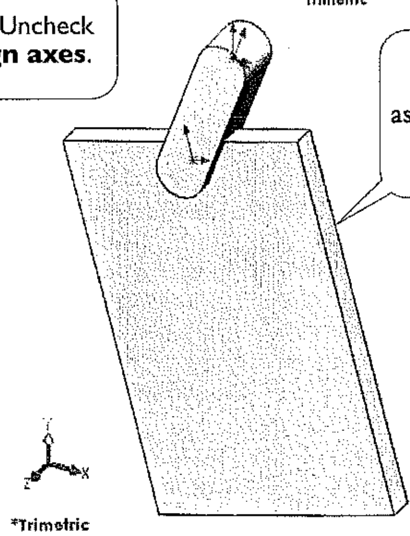
[11] **Concentric1** is created by clicking this cylindrical face...

[14] The **Link's** inner frontal face.

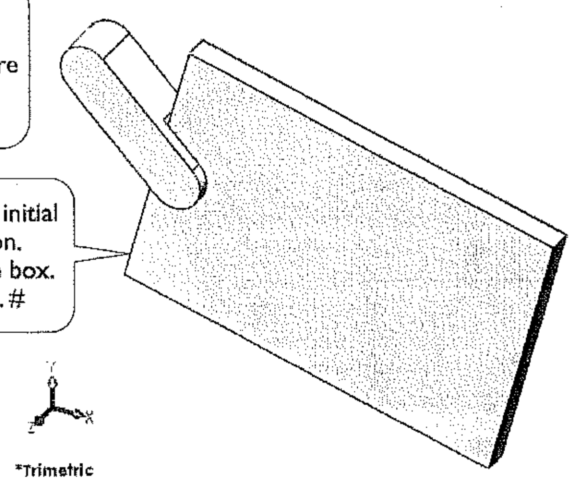
[12] And this cylindrical face.



[17] Uncheck **Align axes**.

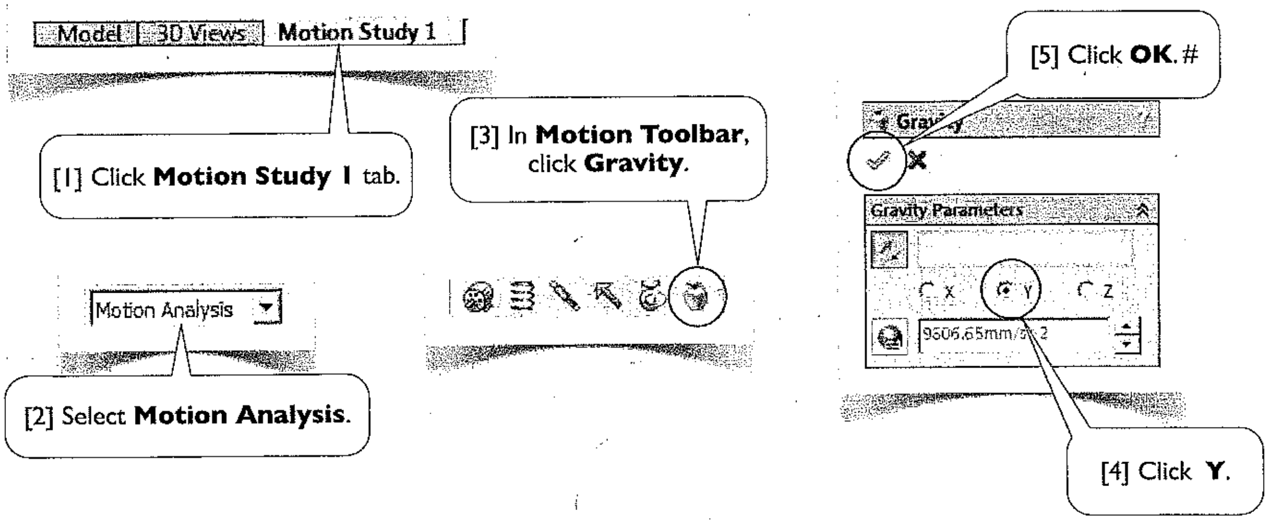


[19] Now, use your mouse to test the assembly and make sure all the **Mates** are correct.

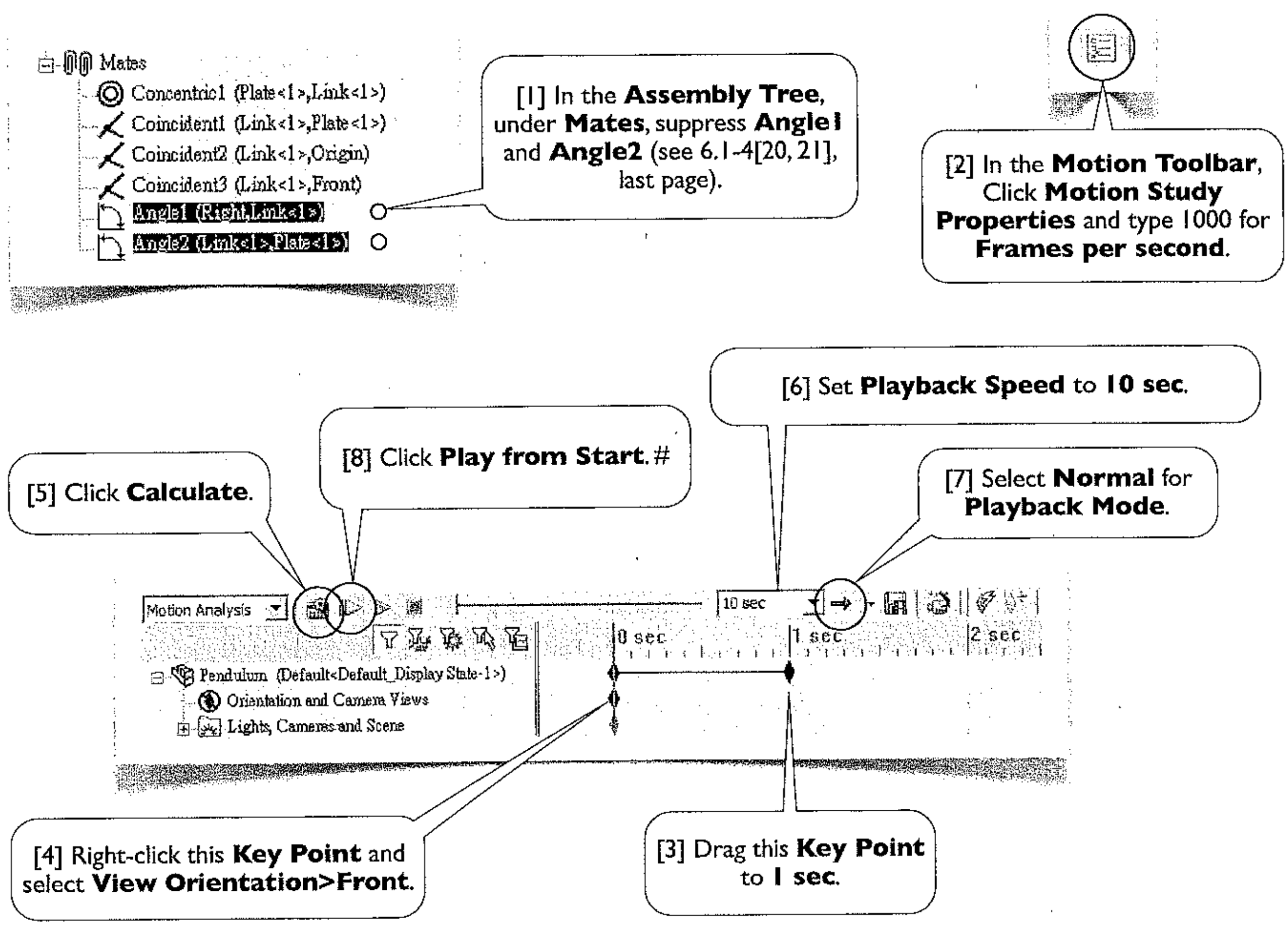


[22] This is the initial configuration. Dismiss **Mate** box. Save the file. #

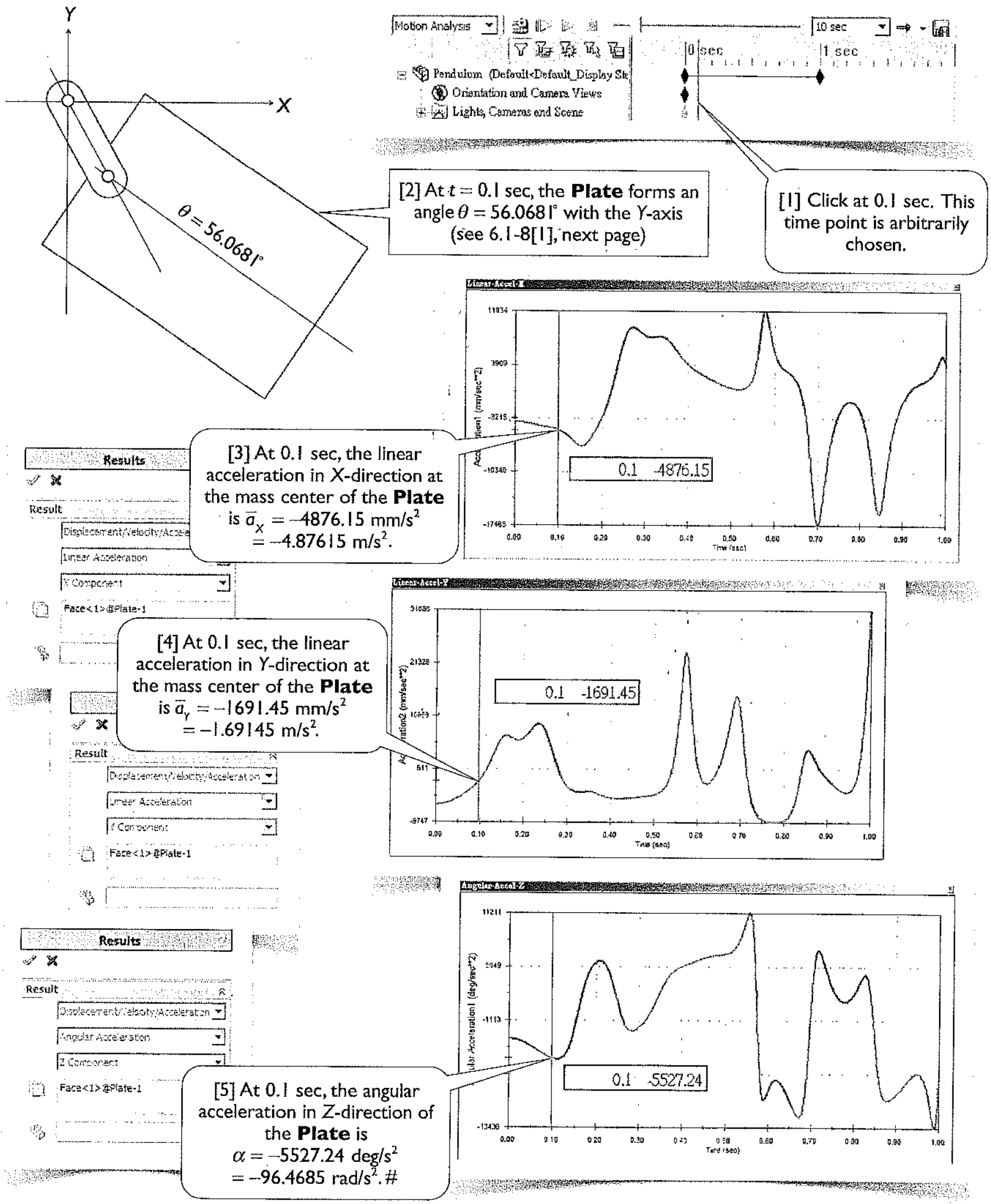
6.1-5 Create a **Motion Study** and Set Up **Gravity**



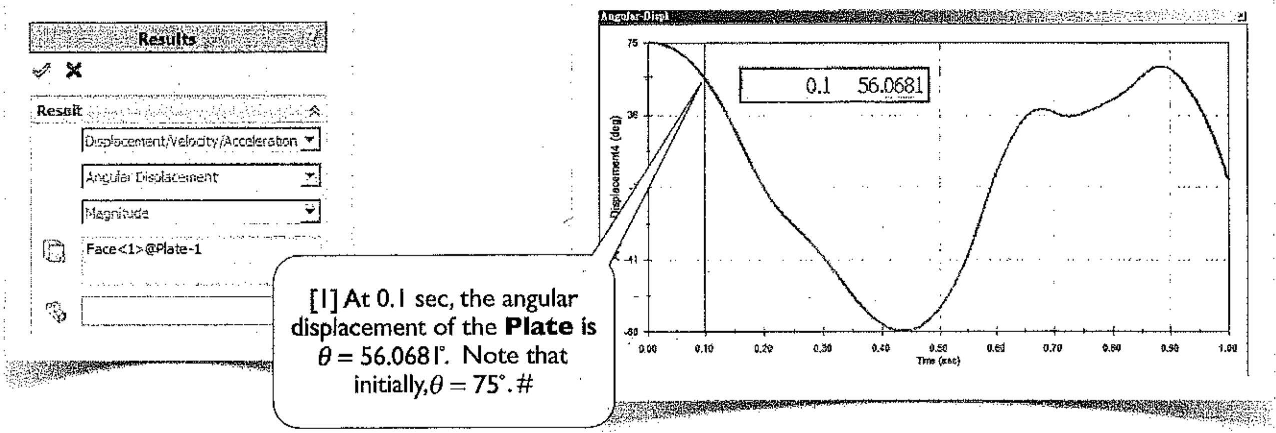
6.1-6 Calculate and Animate **Results**



6.1-7 Results: Linear and Angular Accelerations of the **Plate**

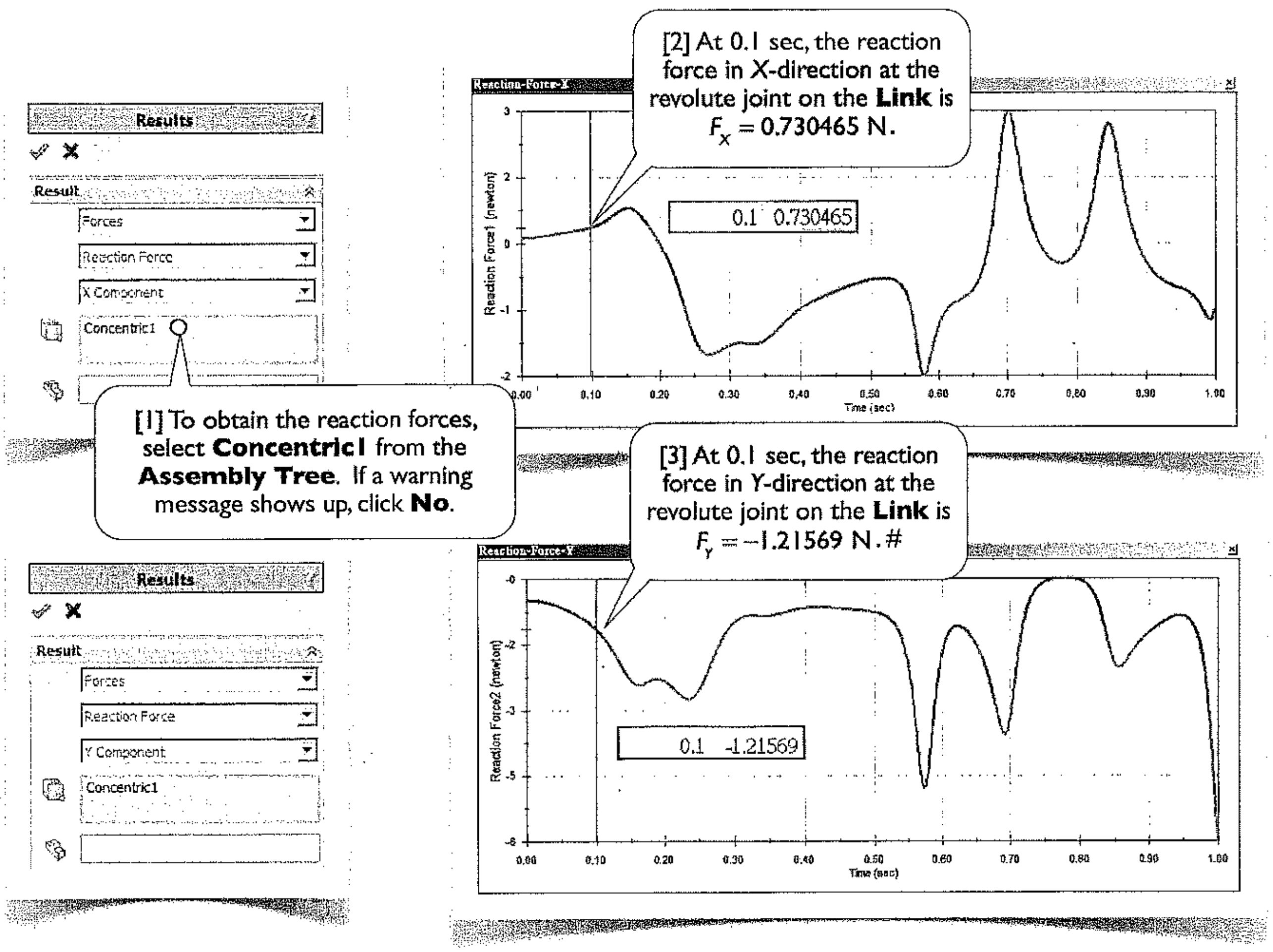


6.1-8 Results: Angular Displacement of the **Plate**



[1] At 0.1 sec, the angular displacement of the **Plate** is $\theta = 56.0681°$. Note that initially, $\theta = 75°$. #

6.1-9 Results: Reaction Forces at **Concentric1**



[1] To obtain the reaction forces, select **Concentric1** from the **Assembly Tree**. If a warning message shows up, click **No**.

[2] At 0.1 sec, the reaction force in X-direction at the revolute joint on the **Link** is $F_X = 0.730465$ N.

[3] At 0.1 sec, the reaction force in Y-direction at the revolute joint on the **Link** is $F_Y = -1.21569$ N. #

6.1-10 Newton's 2nd Law: **Plate**

[1] Newton's 2nd Law for a rigid body in plane motion (Eq. 6.1-1(1), page 122) states that *the external forces and moments acting on a rigid body are equivalent to the* ***effective force and moments*** *acting on the particle. The effective force of a rigid body is simply the product of its mass and the acceleration at the mass center; the effective moment of a rigid body is the product of its moment of inertia $\bar{I}$ and the angular acceleration α.*

The external forces acting on the **Plate** and the effective forces and moment on the **Plate** are shown in [2-8]. It's easy to confirm that these forces and moments indeed satisfy Newton's 2nd Law; i.e., in X-direction,

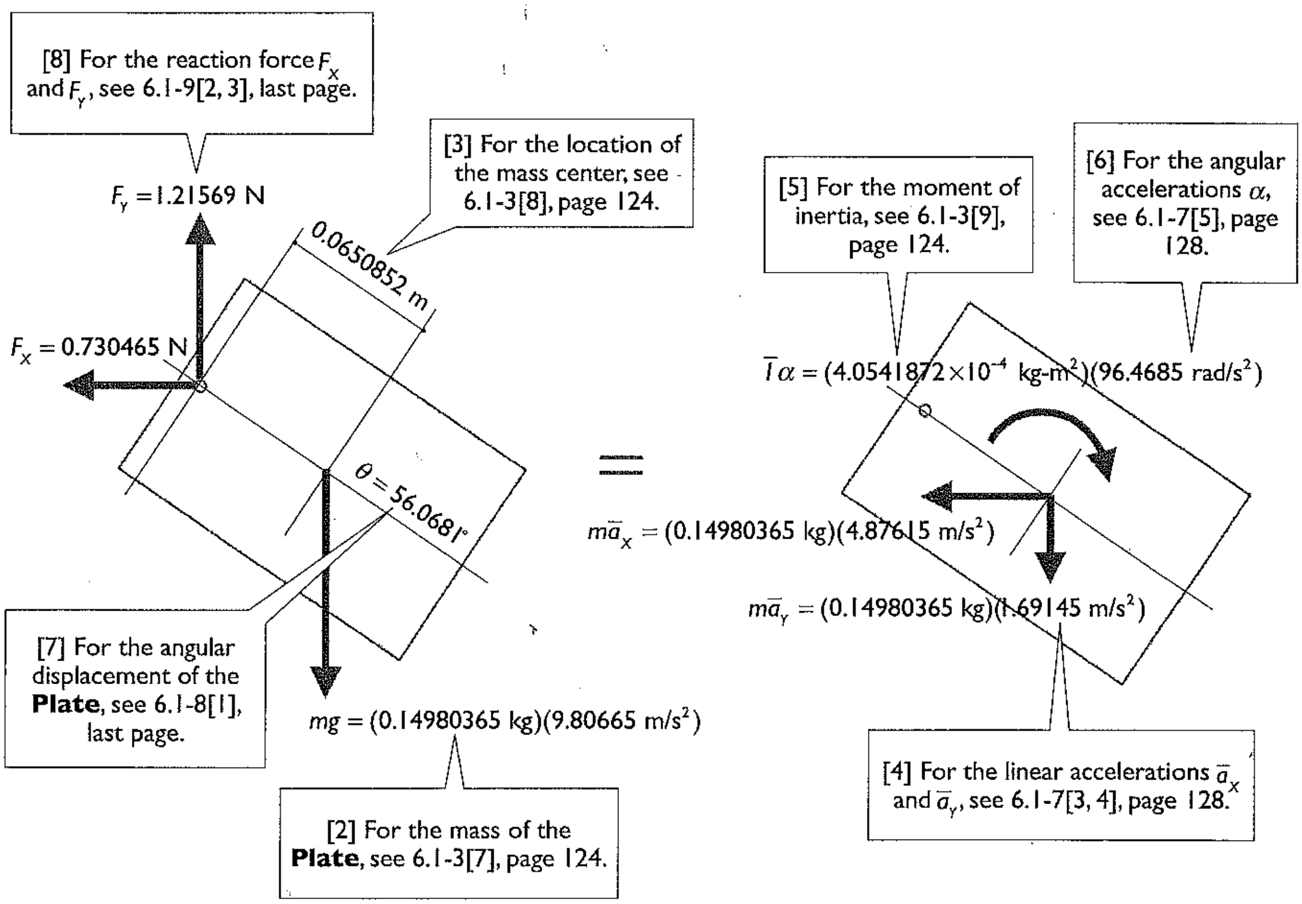
$$-0.730465 \approx (0.14980365)(-4.87615)$$

In Y-direction,

$$1.21569-(0.14980365)(9.80665) \approx (0.14980365)(-1.69145)$$

Taking the moment about the mass center, we have

$$(0.730465)(0.0650852\cos 56.0681^\circ)-(1.21569)(0.0650852\sin 56.0681^\circ) \approx -(4.0541872\times 10^{-4})(96.4685)$$



Wrap Up

[9] Save all files and exit **SOLIDWORKS**. #