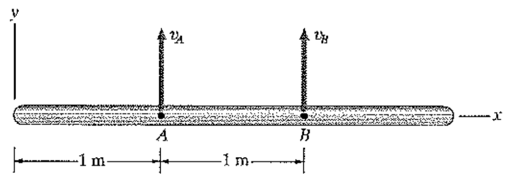
Problem 17.64 If the bar has a clockwise angular velocity of 10 rad/s and $v_A = 20$ m/s, what are the coordinates of its instantaneous center of the bar, and what is the value of v_B?



Solution: Assume that the coordinates of the instantaneous center are (x_C, y_C), $\boldsymbol{\omega} = -\omega\mathbf{k} = -10\mathbf{k}$. The distance to point A is $\mathbf{r}_{A/C} = (1 - x_C)\mathbf{i} + y_C\mathbf{j}$. The velocity at A is

$$\mathbf{v}_A = 20\mathbf{j} = \boldsymbol{\omega} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega \\ 1 - x_C & y_C & 0 \end{bmatrix}$$

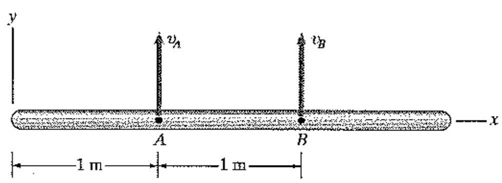
$$= y_C\omega\mathbf{i} - \omega(1 - x_C)\mathbf{j},$$

from which $y_C\omega\mathbf{i} = 0$, and $(20 + \omega(1 - x_C))\mathbf{j} = 0$.

Substitute $\omega = 10$ rad/s to obtain $y_C = 0$ and $x_C = 3$ m. The coordinates of the instantaneous center are $\boxed{(3, 0)\ (\mathrm{m})}$. The vector distance from C to B is $\mathbf{r}_{B/C} = (2 - 3)\mathbf{i} = -\mathbf{i}$ (m). The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -10 \\ -1 & 0 & 0 \end{bmatrix} = -10(-\mathbf{j}) \boxed{= 10\mathbf{j}\ (\mathrm{m/s})}$$

Problem 17.65 In Problem 17.64, if $v_A = 24$ m/s and $v_B = 36$ m/s, what are the coordinates of the instantaneous center of the bar, and what is its angular velocity?



Solution: Let (x_C, y_C) be the coordinates of the instantaneous center. The vectors from the instantaneous center and the points A and B are $\mathbf{r}_{A/C} = (1 - x_C)\mathbf{i} + y_C\mathbf{j}$ (m) and $\mathbf{r}_{B/C} = (2 - x_C)\mathbf{i} + y_C\mathbf{j}$. The velocity of A is given by

$$\mathbf{v}_A = 24\mathbf{j} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 1 - x_C & y_C & 0 \end{bmatrix}$$

$$= -\omega_{AB}y_C\mathbf{i} + \omega_{AB}(1 - x_C)\mathbf{j}\ (\mathrm{m/s})$$

The velocity of B is

$$\mathbf{v}_B = 36\mathbf{j} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 2 - x_C & y_C & 0 \end{bmatrix}$$

$$= -y_C\omega_{AB}\mathbf{i} + \omega_{AB}(2 - x_C)\mathbf{j}\ (\mathrm{m/s}).$$

Separate components:

$$24 - \omega_{AB}(1 - x_C) = 0,$$

$$36 - \omega_{AB}(2 - x_C) = 0,$$

$$\omega_{AB}y_C = 0.$$

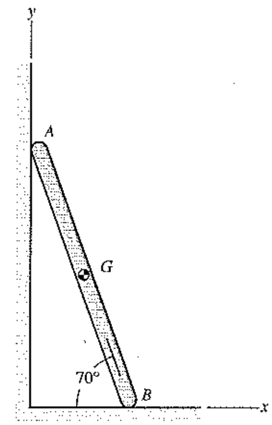
Solve: $\boxed{x_C = -1,}$ $\boxed{y_C = 0,}$

and $\boxed{\omega_{AB} = 12\ \mathrm{rad/s}}$ *counter clockwise.*

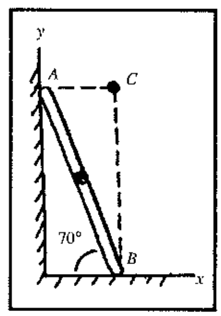
Problem 17.67 Points A and B of the 1-m bar slide on the plane surfaces. The velocity of B is $\mathbf{v}_B = 2\mathbf{i}$ (m/s).

(a) What are the coordinates of the instantaneous center of the bar?

(b) Use the instantaneous center to determine the velocity at A.



Solution:



(a) A is constrained to move parallel to the y axis, and B is constrained to move parallel to the x axis. Draw perpendiculars to the velocity vectors at A and B. From geometry, the perpendiculars intersect at

$\boxed{(\cos 70^\circ, \sin 70^\circ) = (0.3420, 0.9397) \text{ m}}$.

(b) The vector from the instantaneous center to point B is

$$\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = 0.3420\mathbf{i} - (0.3420\mathbf{i} + 0.9397\mathbf{j}) = -0.9397\mathbf{j}$$

The angular velocity of bar AB is obtained from

$$\mathbf{v}_B = 2\mathbf{i} = \omega_{AB} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 0 & -0.9397 & 0 \end{bmatrix}$$

$$= \omega_{AB}(0.9397)\mathbf{i},$$

from which $\omega_{AB} = \dfrac{2}{0.9397} = 2.13$ rad/s.

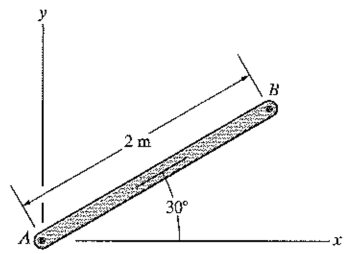
The vector from the instantaneous center to point A is $\mathbf{r}_{A/C} = \mathbf{r}_A - \mathbf{r}_C = -0.3420\mathbf{i}$ (m). The velocity at A is

$$\mathbf{v}_A = \omega_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.13 \\ -0.3420 & 0 & 0 \end{bmatrix}$$

$$\boxed{= -0.7279\mathbf{j} \text{ (m/s)}.}$$

Problem 17.69 Point A of the bar is moving at 8 m/s in the direction of the unit vector $0.966\mathbf{i} - 0.259\mathbf{j}$, and point B is moving in the direction of the unit vector $0.766\mathbf{i} + 0.643\mathbf{j}$.

(a) What are the coordinates of the bar's instantaneous center?
(b) What is the bar's angular velocity?



Solution: Assume the instantaneous center Q is located at (x, y).

Then

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_{A/Q}, \quad \mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/Q}$$

$$(8 \text{ m/s})(0.966\mathbf{i} - 0.259\mathbf{j}) = \omega\mathbf{k} \times (-x\mathbf{i} - y\mathbf{j})$$

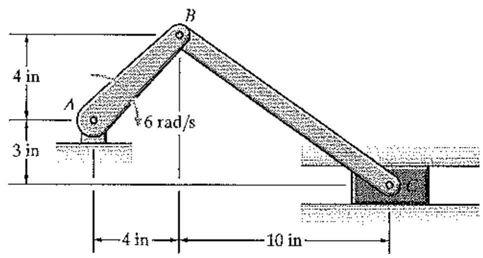
$$v_B(0.766\mathbf{i} + 0.643\mathbf{j}) = \omega\mathbf{k} \times ([\{2 \text{ m}\}\cos 30^\circ - x]\mathbf{i}$$

$$+ [\{2 \text{ m/s}\}\sin 30^\circ - y]\mathbf{j})$$

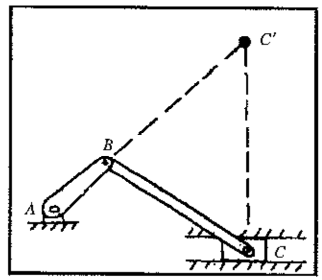
Expanding we have the four equations

$$\left.\begin{array}{l} 7.73 \text{ m/s} = \omega y \\ -2.07 \text{ m/s} = -\omega x \\ 0.766 v_B = \omega(y - 1 \text{ m}) \\ 0.643 v_B = \omega(1.73 \text{ m} - x) \end{array}\right\} \Rightarrow \boxed{\begin{array}{l} x = 0.623 \text{ m} \\ y = 2.32 \text{ m} \end{array}}$$

Problem 17.75 Bar AB rotates at 6 rad/s in the clockwise direction. Use instantaneous centers to determine the angular velocity of bar BC.



Solution: Choose a coordinate system with origin at A and y axis vertical. Let C' denote the instantaneous center. The instantaneous center for bar AB is the point A, by definition, since A is the point of zero velocity. The vector AB is $\mathbf{r}_{B/A} = 4\mathbf{i} + 4\mathbf{j}$ (in.). The velocity at B is



$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -6 \\ 4 & 4 & 0 \end{bmatrix} = 24\mathbf{i} - 24\mathbf{j} \text{ (in/s)}.$$

The unit vector parallel to AB is also the unit vector perpendicular to the velocity at B,

$$\mathbf{e}_{AB} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}).$$

The vector location of a point on a line perpendicular to the velocity at B is $\mathbf{L}_{AB} = L_{AB}\mathbf{e}_{AB}$, where L_{AB} is the magnitude of the distance from point A to the point on the line. The vector location of a point on a perpendicular to the velocity at C is $\mathbf{L}_C = (14\mathbf{i} + y\mathbf{j})$ where y is the y-coordinate of the point referenced to an origin at A. When the two lines intersect,

$$\frac{L_{AB}}{\sqrt{2}}\mathbf{i} = 14\mathbf{i},$$

and $y = \dfrac{L_{AB}}{\sqrt{2}} = 14$

from which $L_{AB} = 19.8$ in., and the coordinates of the instantaneous center are (14, 14) (in.).

[*Check*: The line AC' is the hypotenuse of a right triangle with a base of 14 in. and interior angles of 45°, from which the coordinates of C' are (14, 14) in. *check*.]. The angular velocity of bar BC is determined from the known velocity at B. The vector from the instantaneous center to point B is

$$\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = 4\mathbf{i} + 4\mathbf{j} - 14\mathbf{i} - 14\mathbf{j} = -10\mathbf{i} - 10\mathbf{j}.$$

The velocity of point B is

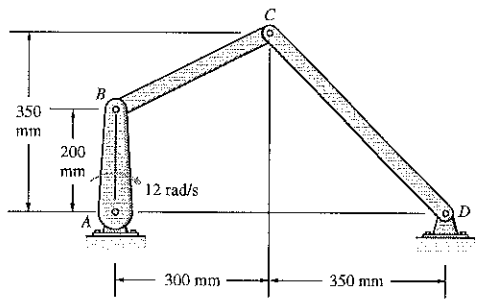
$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -10 & -10 & 0 \end{bmatrix}$$

$$= \omega_{BC}(10\mathbf{i} - 10\mathbf{j}) \text{ (in/s)}.$$

Equate the two expressions for the velocity: $24 = 10\omega_{BC}$, from which

$\boxed{\omega_{BC} = 2.4 \text{ rad/s}}$,

Problem 17.78 Bar AB rotates at 12 rad/s in the clockwise direction. Use instantaneous centers to determine the angular velocities of bars BC and CD.



Solution: Choose a coordinate system with the origin at A and the x axis parallel to AD. The instantaneous center of bar AB is point A, by definition. The velocity of point B is normal to the bar AB. Using the instantaneous center A and the known angular velocity of bar AB the velocity of B is

$$\mathbf{v}_B = \omega \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -12 \\ 0 & 200 & 0 \end{bmatrix} = 2400\mathbf{i} \text{ (mm/s)}.$$

The unit vector perpendicular to the velocity of B is $\mathbf{e}_{AB} = \mathbf{j}$, and a point on a line perpendicular to the velocity at B is $\mathbf{L}_{AB} = L_{AB}\mathbf{j}$ (mm). The instantaneous center of bar CD is point D, by definition. The velocity of point C is constrained to be normal to bar CD. The interior angle at D is 45°, by inspection. The unit vector parallel to DC (and perpendicular to the velocity at C) is

$$\mathbf{e}_{DC} = -\mathbf{i}\cos 45^\circ + \mathbf{j}\sin 45^\circ = \left(\frac{1}{\sqrt{2}}\right)(-\mathbf{i} + \mathbf{j}).$$

The point on a line parallel to DC is

$$\mathbf{L}_{DC} = \left(650 - \frac{L_{DC}}{\sqrt{2}}\right)\mathbf{i} + \frac{L_{DC}}{\sqrt{2}}\mathbf{j} \text{ (mm)}.$$

At the intersection of these lines $\mathbf{L}_{AB} = \mathbf{L}_{DC}$, from which

$$\left(650 - \frac{L_{DC}}{\sqrt{2}}\right) = 0$$

and $L_{AB} = \dfrac{L_{DC}}{\sqrt{2}}$,

from which $L_{DC} = 919.2$ mm, and $L_{AB} = 650$ mm. The coordinates of the instantaneous center of bar BC are (0, 650) (mm). Denote this center by C'. The vector from C' to point B is

$$\mathbf{r}_{B/C'} = \mathbf{r}_B - \mathbf{r}_{C'} = 200\mathbf{j} - 650\mathbf{j} = -450\mathbf{j}.$$

The vector from C' to point C is

$$\mathbf{r}_{C/C'} = 300\mathbf{i} + 350\mathbf{j} - 650\mathbf{j} = 300\mathbf{i} - 300\mathbf{j} \text{ (mm)}.$$

The velocity of point B is

$$\mathbf{v}_B = \omega_{BC} \times \mathbf{r}_{B/C'} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 0 & -450 & 0 \end{bmatrix} = 450\omega_{BC}\mathbf{i} \text{ (mm/s)}.$$

Equate and solve: $2400 = 450\omega_{BC}$, from which

$$\boxed{\omega_{BC} = \frac{2400}{450} = 5.33 \text{ (rad/s)}}.$$

The angular velocity of bar CD is determined from the known velocity at point C. The velocity at C is

$$\mathbf{v}_C = \omega_{BC} \times \mathbf{r}_{C/C'} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 5.33 \\ 300 & -300 & 0 \end{bmatrix}$$

$$= 1600\mathbf{i} + 1600\mathbf{j} \text{ (mm/s)}.$$

The vector from D to point C is $\mathbf{r}_{C/D} = -350\mathbf{i} + 350\mathbf{j}$ (mm). The velocity at C is

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -350 & 350 & 0 \end{bmatrix}$$

$$= -350\omega_{CD}\mathbf{i} - 350\omega_{CD}\mathbf{j} \text{ (mm/s)}.$$

Equate and solve: $\boxed{\omega_{CD} = -4.57 \text{ rad/s}}$ clockwise.