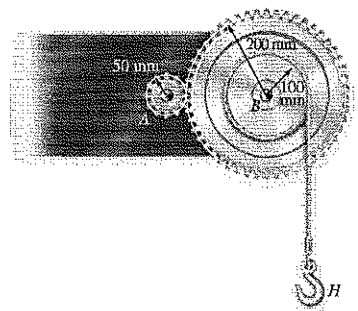
Problem 17.1 In Active Example 17.1, suppose that at a given instant the hook H is moving downward at 2 m/s. What is the angular velocity of gear A at that instant?



Solution: The angular velocity of gear B is

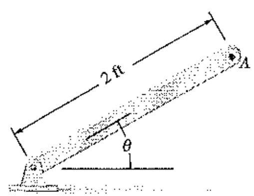
$$\omega_B = \frac{v_H}{r_H} = \frac{2\text{ m/s}}{0.1\text{ m}} = 20\text{ rad/s}.$$

The gears are connected through the common velocity of the contact points

$$r_B\omega_B = r_A\omega_A \Rightarrow \omega_A = \frac{r_B}{r_A}\omega_B = \frac{0.2\text{ m}}{0.05\text{ m}}(20\text{ rad/s}) = 80\text{ rad/s}.$$

$\boxed{\omega_A = 80\text{ rad/s counterclockwise.}}$

Problem 17.2 The angle θ (in radians) is given as a function of time by $\theta = 0.2\pi t^2$. At $t = 4$ s, determine the magnitudes of (a) the velocity of point A and (b) the tangential and normal components of acceleration of point A.



Solution: We have

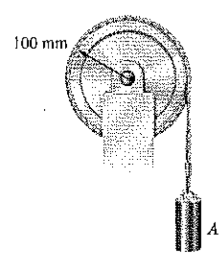
$$\theta = 0.2\pi t^2, \quad \omega = \frac{d\theta}{dt} = 0.4\pi t, \quad \alpha = \frac{d\omega}{dt} = 0.4\pi.$$

Then

(a) $v = r\omega = (2)(0.4\pi)(4) = 10.1$ ft/s. $\boxed{v = 10.1\text{ ft/s.}}$

(b) $a_n = r\omega^2 = (2)[(0.4\pi)(4)]^2 = 50.5\text{ ft/s}^2,$
$a_t = r\alpha = (2)(0.4\pi) = 2.51\text{ ft/s}^2.$ $\boxed{a_n = 50.5\text{ ft/s}^2,\ a_t = 2.51\text{ ft/s}^2.}$

Problem 17.3 The mass A starts from rest at $t = 0$ and falls with a constant acceleration of 8 m/s^2. When the mass has fallen one meter, determine the magnitudes of (a) the angular velocity of the pulley and (b) the tangential and normal components of acceleration of a point at the outer edge of the pulley.



Solution: We have

$$a = 8\text{ m/s}^2, v = \sqrt{2as} = \sqrt{2(8\text{ m/s}^2)(1\text{ m})} = 4\text{ m/s},$$

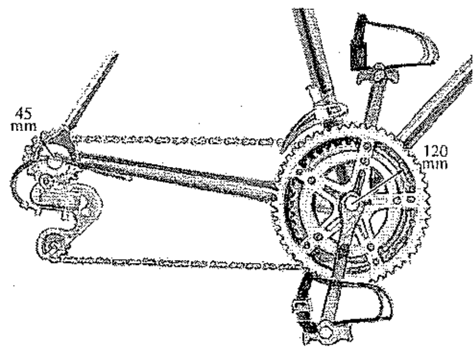
$$\omega = \frac{v}{r} = \frac{4\text{ m/s}}{0.1\text{ m}} = 40\text{ rad/s},$$

$$\alpha = \frac{a}{r} = \frac{8\text{ m/s}^2}{0.1\text{ m}} = 80\text{ rad/s}^2.$$

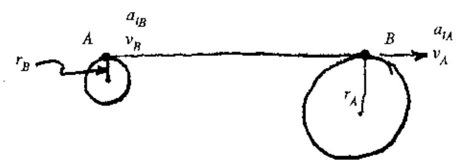
(a) $\boxed{\omega = 40\text{ rad/s.}}$

(b) $a_t = r\alpha = (0.1\text{ m})(80\text{ rad/s}^2) = 8\text{ m/s}^2,$
$a_n = r\omega^2 = (0.1\text{ m})(40\text{ rad/s})^2 = 160\text{ m/s}^2.$ $\boxed{a_t = 8\text{ m/s}^2,\ a_n = 160\text{ m/s}^2.}$

Problem 17.6 (a) If the bicycle's 120-mm sprocket wheel rotates through one revolution, through how many revolutions does the 45-mm gear turn? (b) If the angular velocity of the sprocket wheel is 1 rad/s, what is the angular velocity of the gear?



Solution: The key is that the tangential accelerations and tangential velocities along the chain are of constant magnitude



(a) $\theta_B = 2.67$ rev

(b) $v_B = r\omega_B \qquad v_A = r_A\omega_A$

$v_A = v_B$

$v_B = (0.045)\omega_B \qquad v_A = (0.120)(1)$

$$\omega_B = \left(\frac{120}{45}\right) \text{ rad/s} = 2.67 \text{ rad/s}$$

$$r_B\frac{d\theta_B}{dt} = r_A\frac{d\theta_A}{dt}$$

Integrating, we get

$r_B\theta_B = r_A\theta_A \qquad r_A = 0.120$ m

$r_B = 0.045$ m

$$\theta_B = \left(\frac{120}{45}\right)(1) \text{ rev} \qquad \theta_A = 1 \text{ rev.}$$

Problem 17.7 The rear wheel of the bicycle in Problem 17.6 has a 330-mm radius and is rigidly attached to the 45-mm gear. It the rider turns the pedals, which are rigidly attached to the 120-mm sprocket wheel, at one revolution per second, what is the bicycle's velocity?

Solution: The angular velocity of the 120 mm sprocket wheel is $\omega = 1$ rev/s $= 2\pi$ rad/s. Use the solution to Problem 17.6. The angular velocity of the 45 mm gear is

$$\omega_{45} = 2\pi\left(\frac{120}{45}\right) = 16.76 \text{ rad/s.}$$

This is also the angular velocity of the rear wheel, from which the velocity of the bicycle is

$v = \omega_{45}(330) = 5.53$ m/s.

Problem 17.12 Consider the bar shown in Problem 17.11. If $|\mathbf{v}_A| = 3$ m/s and $|\mathbf{a}_A| = 28$ m/s^2, what are $|\mathbf{v}_B|$ and $|\mathbf{a}_B|$?

Solution:

$$v_A = \omega r \Rightarrow 3 \text{ m/s} = \omega\sqrt{(0.4)^2 + (0.4)^2} \text{ m} \Rightarrow \omega = 5.30 \text{ rad/s}$$

$$a_{An} = \omega^2 r = (5.3 \text{ rad/s})^2\sqrt{(0.4)^2 + (0.4)^2} \text{ m} = 15.9 \text{ m/s}^2$$

$$a_{At} = \sqrt{a_A{}^2 - a_{An}{}^2} = \sqrt{(28)^2 - (15.9)^2} \text{ m/s}^2 = 23.0 \text{ m/s}^2$$

$$a_{At} = \alpha r \Rightarrow 23.0 \text{ m/s}^2 = \alpha\sqrt{(0.4)^2 + (0.4)^2} \text{ m} \Rightarrow \alpha = 40.7 \text{ rad/s}$$

$$\boxed{v_B = \omega\sqrt{(0.4)^2 + (-0.2)^2} \text{ m} = 2.37 \text{ m/s}}$$

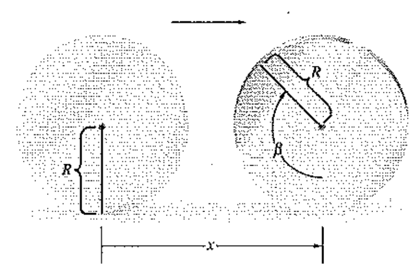
$$a_{Bt} = \alpha\sqrt{(0.4)^2 + (-0.2)^2} = 18.2 \text{ m/s}^2$$

$$a_{Bn} = \omega^2\sqrt{(0.4)^2 + (-0.2)^2} = 12.6 \text{ m/s}^2$$

$$\boxed{a_B = \sqrt{(18.2)^2 + (12.6)^2} \text{ m/s}^2 = 22.1 \text{ m/s}^2}$$

Problem 17.13 A disk of radius $R = 0.5$ m rolls on a horizontal surface. The relationship between the horizontal distance x the center of the disk moves and the angle β through which the disk rotates is $x = R\beta$. Suppose that the center of the disk is moving to the right with a constant velocity of 2 m/s.

(a) What is the disk's angular velocity?
(b) Relative to a nonrotating reference frame with its origin at the center of the disk, what are the magnitudes of the velocity and acceleration of a point on the edge of the disk?

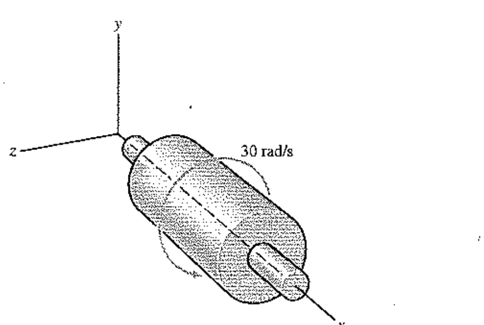


Solution:

(a) $x = R\beta \Rightarrow \dot{x} = R\dot{\beta} \Rightarrow v = R\omega \Rightarrow \boxed{\omega = \frac{v}{R} = \frac{2 \text{ m/s}}{0.5 \text{ m}} = 4 \text{ rad/s}}$

(b) $$\boxed{\begin{aligned} v &= R\omega = (0.5 \text{ m})(4 \text{ rad/s}) = 2 \text{ m/s} \\ a &= a_n = R\omega^2 = (0.5 \text{ m})(4 \text{ rad/s})^2 = 8 \text{ m/s}^2 \end{aligned}}$$

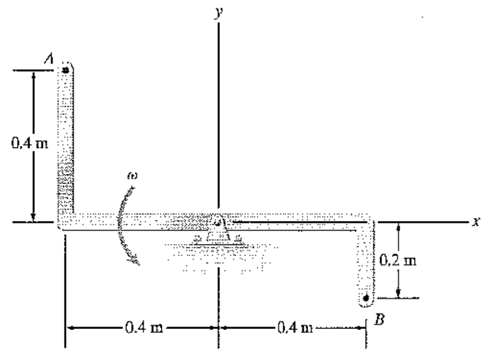
Problem 17.14 The turbine rotates relative to the coordinate system at 30 rad/s about a fixed axis coincident with the x axis. What is its angular velocity vector?



Solution: The angular velocity vector is parallel to the x axis, with magnitude 30 rad/s. By the right hand rule, the positive direction coincides with the positive direction of the x axis.

$\boxed{\omega = 30\mathbf{i}}$ (rad/s).

Problem 17.19 The bar is rotating in the counterclockwise direction with angular velocity ω. The magnitude of the velocity of point A is 6 m/s. Determine the velocity of point B.



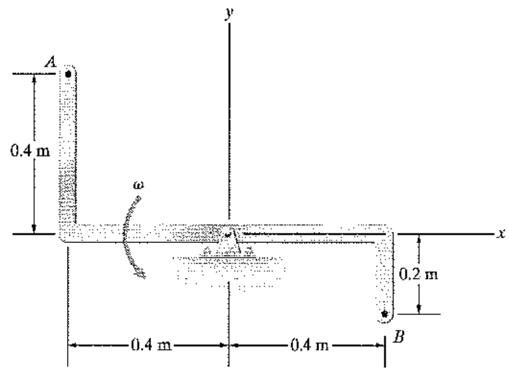
Solution:

$$\omega = \frac{v}{r} = \frac{6 \text{ m/s}}{\sqrt{2}(0.4 \text{ m})} = 10.6 \text{ rad/s}.$$

$$\mathbf{v}_B = \omega \times \mathbf{r}_B = (10.6 \text{ rad/s})\mathbf{k} \times (0.4\mathbf{i} - 0.2\mathbf{j}) \text{ m}$$

$$\boxed{\mathbf{v}_B = (2.12\mathbf{i} + 4.24\mathbf{j}) \text{ m/s.}}$$

Problem 17.20 The bar is rotating in the counterclockwise direction with angular velocity ω. The magnitude of the velocity of point A relative to point B is 6 m/s. Determine the velocity of point B.



Solution:

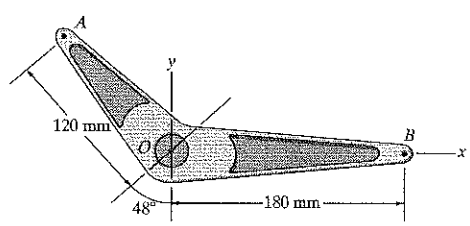
$$r_{A/B} = \sqrt{(0.8 \text{ m})^2 + (0.6 \text{ m})^2} = 1 \text{ m}$$

$$\omega = \frac{v}{r_{A/B}} = \frac{6 \text{ m/s}}{1 \text{ m}} = 6 \text{ rad/s}.$$

$$\mathbf{v}_B = \omega \times \mathbf{r}_B = (6 \text{ rad/s})\mathbf{k} \times (0.4\mathbf{i} - 0.2\mathbf{j}) \text{ m}$$

$$\boxed{\mathbf{v}_B = (1.2\mathbf{i} + 2.4\mathbf{j}) \text{ m/s.}}$$

Problem 17.21 The bracket is rotating about point O with counterclockwise angular velocity ω. The magnitude of the velocity of point A relative to point B is 4 m/s. Determine ω.



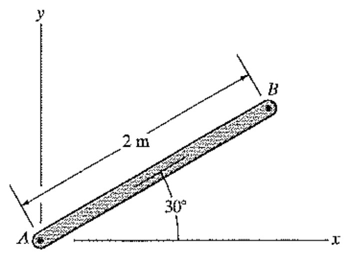
Solution:

$$r_{B/A} = \sqrt{(0.18 + 0.12\cos 48^\circ)^2 + (0.12 \sin 48^\circ)^2} = 0.275 \text{ m}$$

$$\omega = \frac{v_{B/A}}{r_{B/A}} = \frac{4 \text{ m/s}}{0.275 \text{ m}} = 14.5 \text{ rad/s}.$$

$$\boxed{\omega = 14.5 \text{ rad/s.}}$$

Problem 17.29 The bar is moving in the $x-y$ plane and is rotating in the counterclockwise direction. The velocity of point A relative to the reference frame is $\mathbf{v}_A = 12\mathbf{i} - 2\mathbf{j}$ (m/s). The magnitude of the velocity of point A relative to point B is 8 m/s. what is the velocity of point B relative to the reference frame?



Solution:

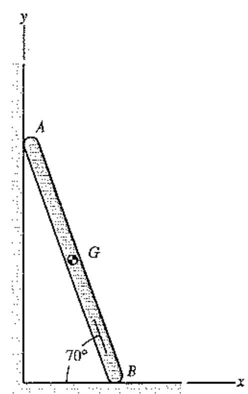
$$\omega = \frac{v}{r} = \frac{8 \text{ m/s}}{2 \text{ m}} = 4 \text{ rad/s},$$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A} = (12\mathbf{i} - 2\mathbf{j}) \text{ m/s} + (4 \text{ rad/s})\mathbf{k} \times (2 \text{ m})(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j})$$

$$\boxed{\mathbf{v}_B = (8\mathbf{i} + 4.93\mathbf{j}) \text{ m/s.}}$$

Problem 17.30 Points A and B of the 2-m bar slide on the plane surfaces. Point B is moving to the right at 3 m/s. What is the velocity of the midpoint G of the bar?

Strategy: First apply Eq. (17.6) to points A and B to determine the bar's angular velocity. Then apply Eq. (17.6) to points B and G.



Solution: Take advantage of the constraints (B stays on the floor, A stays on the wall)

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$v_A\mathbf{j} = (3 \text{ m/s})\mathbf{i} + \omega\mathbf{k} \times (2 \text{ m})(-\cos 70^\circ \mathbf{i} + \sin 70^\circ \mathbf{j})$$

$$= (3 - 1.88\,\omega)\mathbf{i} + (-0.684\,\omega)\mathbf{j}$$

Equating $\mathbf{i}$ components we find $3 - 1.88\,\omega = 0 \Rightarrow \omega = 1.60$ rad/s

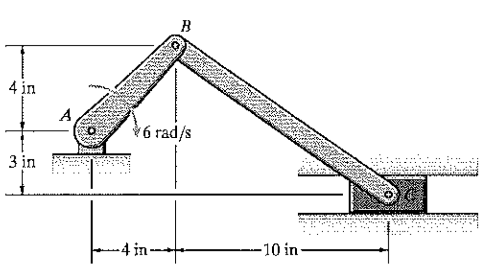
Now find the velocity of point G

$$\mathbf{v}_G = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{G/B}$$

$$= (3 \text{ m/s})\mathbf{i} + (1.60 \text{ rad/s})\mathbf{k} \times (1 \text{ m})(-\cos 70^\circ \mathbf{i} + \sin 70^\circ \mathbf{j})$$

$$\boxed{\mathbf{v}_G = (1.5\mathbf{i} - 0.546\mathbf{j}) \text{ m/s}}$$

Problem 17.31 Bar AB rotates at 6 rad/s in the clockwise direction. Determine the velocity (in in/s) of the slider C.



Solution:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\mathbf{v}_B = 0 - (6 \text{ rad/s})\mathbf{k} \times (4\mathbf{i} + 4\mathbf{j}) \text{ in}$$

$$\mathbf{v}_B = (24\mathbf{i} - 24\mathbf{j}) \text{ in/s.}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$\mathbf{v}_C = (24\mathbf{i} - 24\mathbf{j}) + \omega_{BC}\mathbf{k} \times (10\mathbf{i} - 7\mathbf{j})$$

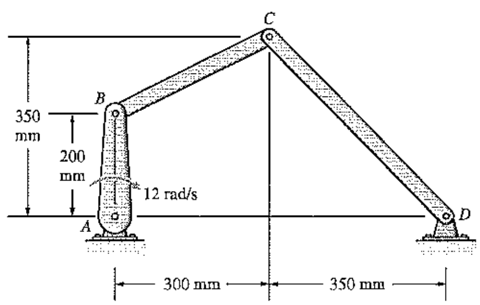
$$\mathbf{v}_C = (24 + 7\,\omega_{BC})\mathbf{i} + (-24 + 10\,\omega_{BC})\mathbf{j}$$

Slider C cannot move in the $\mathbf{j}$ direction, therefore

$$\omega_{BC} = \frac{24}{10} = 2.4 \text{ rad/s.}$$

$$\mathbf{v}_C = (24 + 7(2.4))\mathbf{i} = (43.8 \text{ in/s})\mathbf{i} \quad \boxed{v_C = 43.8 \text{ in/s to the right.}}$$

Problem 17.37 Bar AB rotates at 12 rad/s in the clockwise direction. Determine the angular velocities of bars BC and CD.



Solution: The strategy is analogous to that used in Problem 17.36. The radius vector AB is $\mathbf{r}_{B/A} = 200\mathbf{j}$ (mm). The angular velocity of AB is $\omega = -12\mathbf{k}$ (rad/s). The velocity of point B is

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A} = 0 + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -12 \\ 0 & 200 & 0 \end{bmatrix} = 2400\mathbf{i} \text{ (mm/s)}.$$

The radius vector BC is $\mathbf{r}_{C/B} = 300\mathbf{i} + (350 - 200)\mathbf{j} = 300\mathbf{i} + 150\mathbf{j}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 300 & 150 & 0 \end{bmatrix}$$

$$= (2400 - 150\omega_{BC})\mathbf{i} + \omega_{BC}300\mathbf{j} \text{ (mm/s)}.$$

The radius vector DC is $\mathbf{r}_{C/D} = -350\mathbf{i} + 350\mathbf{j}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_D + \omega_{CD} \times \mathbf{r}_{C/D} = 0 + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -350 & 350 & 0 \end{bmatrix}$$

$$= -350\omega_{CD}(\mathbf{i} + \mathbf{j}).$$

Equate the two expressions for $\mathbf{v}_C$, and separate components:

$$(2400 - 150\omega_{BC} + 350\omega_{CD})\mathbf{i} = 0,$$

and $(300\omega_{BC} + 350\omega_{CD})\mathbf{j} = 0.$

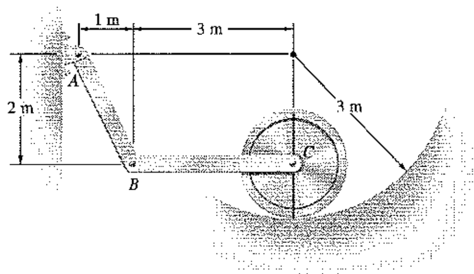
Solve: $\omega_{BC} = 5.33$ rad/s,

$\boxed{\omega_{BC} = 5.33\mathbf{k} \text{ (rad/s)}}$.

$\omega_{CD} = -4.57$ rad/s,

$\boxed{\omega_{CD} = -4.57\mathbf{k} \text{ (rad/s)}}$.

Problem 17.38 Bar AB is rotating at 10 rad/s in the counterclockwise direction. The disk rolls on the circular surface. Determine the angular velocities of bar BC and the disk at the instant shown.



Solution: The point "D" at the bottom of the wheel has zero velocity.

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$= 0 + (10)\mathbf{k} \times (1\mathbf{i} - 2\mathbf{j}) = (20\mathbf{i} + 10\mathbf{j}) \text{ m/s.}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= (20\mathbf{i} + 10\mathbf{j}) + \omega_{BC}\mathbf{k} \times (3\mathbf{i}) = (20)\mathbf{i} + (10 + 3\omega_{BC})\mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$$

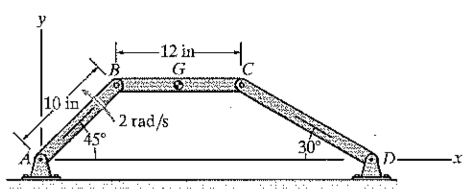
$$= (20)\mathbf{i} + (10 + 3\omega_{BC})\mathbf{j} + \omega_{CD}\mathbf{k} \times (-1\mathbf{j}) = (20 + \omega_{CD})\mathbf{i} + (10 + 3\omega_{BC})\mathbf{j}$$

Since the velocity of D is zero, we can set the components of velocity equal to zero and solve to find

$$\omega_{\text{disk}} = \omega_{CD} = -20 \text{ rad/s}, \quad \omega_{BC} = 3.33 \text{ rad/s.}$$

$\boxed{\omega_{\text{disk}} = 20 \text{ rad/s clockwise}, \quad \omega_{BC} = 3.33 \text{ rad/s clockwise.}}$

Problem 17.39 Bar AB rotates at 2 rad/s in the counterclockwise direction. Determine the velocity of the midpoint G of bar BC.



Solution: We first need to find the angular velocities of BC and CD

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (2 \text{ rad/s})\mathbf{k} \times (10 \text{ in.})(\cos 45^\circ\mathbf{i} + \sin 45^\circ\mathbf{j})$$

$$= (-14.1\mathbf{i} + 14.1\mathbf{j}) \text{ in./s}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = (-14.1\mathbf{i} + 14.1\mathbf{j}) \text{ in./s} + \omega_{BC}\mathbf{k} \times (12 \text{ in.})\mathbf{i}$$

$$= [(-14.1 \text{ in./s})\mathbf{i} + (14.1 \text{ in./s} + \{12 \text{ in.}\}\omega_{BC})\mathbf{j}]$$

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$$

$$= [(-14.1 \text{ in./s})\mathbf{i} + (14.1 \text{ in./s} + \{12 \text{ in.}\}\omega_{BC})\mathbf{j}]$$

$$+ \omega_{CD}\mathbf{k} \times (10 \text{ in.})[\sin 45^\circ \cot 30^\circ\mathbf{i} - \sin 45^\circ\mathbf{j}]$$

$$= [(-14.1 \text{ in./s} + \{7.07 \text{ in.}\}\omega_{CD})\mathbf{i} + (14.1 \text{ in./s}$$

$$+ \{12 \text{ in.}\}\omega_{BC} + \{12.2 \text{ in.}\}\omega_{CD})\mathbf{j}]$$

Since D is fixed, we set both components to zero and solve for the angular velocities

$$\begin{array}{l} -14.1 \text{ in./s} + \{7.07 \text{ in.}\}\omega_{CD} = 0 \\ 14.1 \text{ in./s} + \{12 \text{ in.}\}\omega_{BC} + \{12.2 \text{ in.}\}\omega_{CD} = 0 \end{array} \Rightarrow \begin{array}{l} \omega_{BC} = -3.22 \text{ rad/s} \\ \omega_{CD} = 2.00 \text{ rad/s} \end{array}$$

Now we can find the velocity of point G.

$$\mathbf{v}_G = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{G/B}$$

$$= (-14.1\mathbf{i} + 14.1\mathbf{j}) \text{ in./s} + (-3.22 \text{ rad/s})\mathbf{k} \times (6 \text{ in.})\mathbf{i}$$

$\boxed{\mathbf{v}_G = (-14.1\mathbf{i} - 5.18\mathbf{j}) \text{ in./s}}$